Scheme of Learning

Year 6

#MathsEveryoneCan

White Rose Maths

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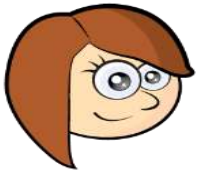
Meet the Characters

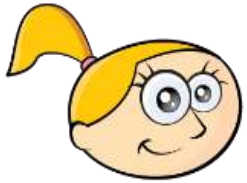
Children love to learn with characters and our team within the scheme will be sure to get them talking and reasoning about mathematical concepts and ideas. Who's your favourite?

Teddy



Rosie

Mo



Eva

Alex

Jack

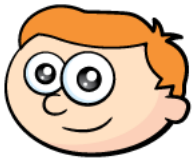
Whitney

Amir

Dora

Tommy

Dexter



Ron

Annie

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number: Place Value		Number: Addition, Subtraction, Multiplication and Division				Number: Fractions				Geometry: Position and Direction	Consolidation
Spring	Number: Decimals		Number: Percentages		Number: Algebra		Measurement: Converting Units	Measurement: Perimeter, Area and Volume		Number: Ratio		Consolidation
Summer	Geometry: Properties of Shape		Problem Solving			Statistics		Investigations				Consolidation

White Rose Maths

Spring - Block 1

Decimals

Overview

Small Steps

- Three decimal places
- Multiply by 10, 100 and 1,000
- Divide by 10, 100 and 1,000
- Multiply decimals by integers
- Divide decimals by integers
- Division to solve problems
- Decimals as fractions
- Fractions to decimals (1)
- Fractions to decimals (2)

NC Objectives

Identify the value of each digit in numbers given to 3 decimal places and multiply numbers by 10, 100 and 1,000 giving answers up to 3 decimal places.

Multiply 1-digit numbers with up to 2 decimal places by whole numbers.

Use written division methods in cases where the answer has up to 2 decimal places.

Solve problems which require answers to be rounded to specified degrees of accuracy.

Three Decimal Places

Notes and Guidance

Children recap their understanding of numbers with up to 3 decimal places. They look at the value of each place value column and describe its value in words and digits.

Children use concrete resources to investigate exchanging between columns e.g. 3 tenths is the same as 30 hundredths.

Mathematical Talk

How many tenths are there in the number? How many hundredths? How many thousandths?

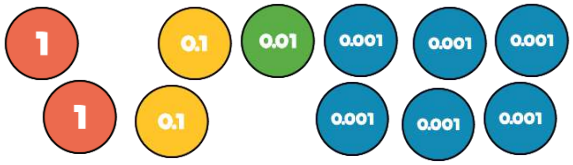
Can you make the number on the place value chart?

How many hundredths are the same as 5 tenths?

What is the value of the zero in this number?

Varied Fluency

- Complete the sentences.



There are ____ ones, ____ tenths, ____ hundredths and ____ thousandths.
The number in digits is _____

- Use counters and a place value chart to represent these numbers.

3.456 | 72.204 | 831.07

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

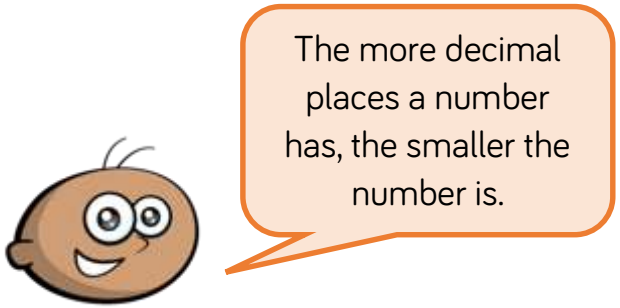
- Write down the value of the 3 in the following numbers.

0.53 362.44 739.8 0.013 3,420.98

Three Decimal Places

Reasoning and Problem Solving

Tommy says,



Do you agree?
Explain why.

Possible answer:

I do not agree with this as the number 4.39 is smaller than the number 4.465, which has more decimal places.

Alex says that 3.24 can be written as 2 ones, 13 tenths and 4 hundredths.

Do you agree?

How can you partition 3.24 starting with 2 ones?
How can you partition 3.24 starting with 1 one?
Think about exchanging between columns.

Possible answer:

I disagree; Alex's numbers would total 3.34. I could make 3.24 by having 2 ones, 12 tenths and 4 hundredths or 1 one, 22 tenths and 4 hundredths.

Four children are thinking of four different numbers.

3.454	4.445
4.345	3.54

Teddy: "My number has four hundredths."

Alex: "My number has the same amount of ones, tenths and hundredths."

Dora: "My number has less ones that tenths and hundredths."

Jack: "My number has 2 decimal places."

Match each number to the correct child.

Teddy: 4.345

Alex: 4.445

Dora: 3.454

Jack: 3.54

Multiply by 10, 100 and 1,000

Notes and Guidance

Children multiply numbers with up to three decimal places by 10, 100 and 1,000
They discover that digits move to the left when they are multiplying and use zero as a place value holder. The decimal point does not move.
Once children are confident in multiplying by 10, 100 and 1,000, they use these skills to investigate multiplying by multiples of these numbers e.g. 2.4 × 20

Mathematical Talk

What number is represented on the place value chart?

Why is 0 important when multiplying by 10, 100 and 1,000?

What patterns do you notice?

What is the same and what is different when multiplying by 10, 100, 1,000 on the place value chart compared with the Gattegno chart?

Varied Fluency

- Identify the number represented on the place value chart.

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths
			● ●	●	

Multiply it by 10, 100 and 1,000 and complete the sentence stem for each.
When multiplied by ____ the counters move ____ places to the _____.

- Use a place value chart to multiply the following decimals by 10, 100 and 1,000

6.4 | 6.04 | 6.004

- Fill in the missing numbers in these calculations

32.4 × ☐ = 324

1.562 × 1,000 = ☐

☐ × 100 = 208

4.3 × ☐ = 86

Multiply by 10, 100 and 1,000

Reasoning and Problem Solving

Using the digit cards 0-9 create a number with up to 3 decimal places e.g. 3.451
Cover the number using counters on your Gattegno chart.

10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

Explore what happens when you multiply your number by 10, then 100, then 1,000

What patterns do you notice?

Children will be able to see how the counter will move up a row for multiplying by 10, two rows for 100 and three rows for 1,000. They can see that this happens to each digit regardless of the value.
For example, 3.451 × 10 becomes 34.51
Each counter moves up a row but stays in the same column.

Dora says,

Do you agree?
Explain your thinking.

Children should explain that when you multiply by 100 the digits move two places to the left.

For example:
0.34 × 100 = 0.3400 is incorrect as 0.34 is the same as 0.3400
Also:
0.34 + 0 + 0 = 0.34

Children show 0.34 × 100 = 34

Divide by 10, 100 and 1,000

Notes and Guidance

Once children understand how to multiply decimals by 10, 100 and 1,000, they can apply this knowledge to division, which leads to converting between units of measure.

It is important that children continue to understand the importance of 0 as a place holder. Children also need to be aware that 2.4 and 2.40 are the same. Similarly, 12 and 12.0 are equivalent.

Mathematical Talk

What happens to the counters/digits when you divide by 10, 100 or 1,000?

Why is zero important when dividing by 10, 100 and 1,000?

What is happening to the value of the digit each time it moves one column to the right?

What are the relationships between tenths, hundredths and thousandths?

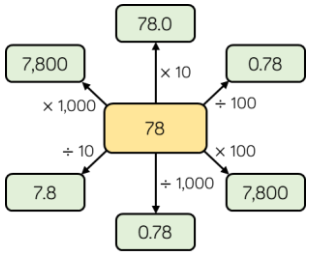
Varied Fluency

- Use the place value chart to divide the following numbers by 10, 100 and 1,000

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

44 | 1.36 | 107 | 5

- Tick the correct answers. Can you explain the mistakes with the incorrect answers?



- Complete the table.

	÷10	÷ 100	÷ 1,000
30			
3 kg			
	0.9		
			9.0
		9.09	

Divide by 10, 100 and 1,000

Reasoning and Problem Solving

Using the following rules, how many ways can you make 70?

- Use a number from column A
- Use an operation from column B.
- Use number from column C.

A	B		C
0.7	×	÷	0.1
7			1
70			10
700			100
7,000			1,000

Possible answers:

0.7 × 100
7 × 10
70 × 1
700 ÷ 10
7,000 ÷ 100
70 ÷ 1

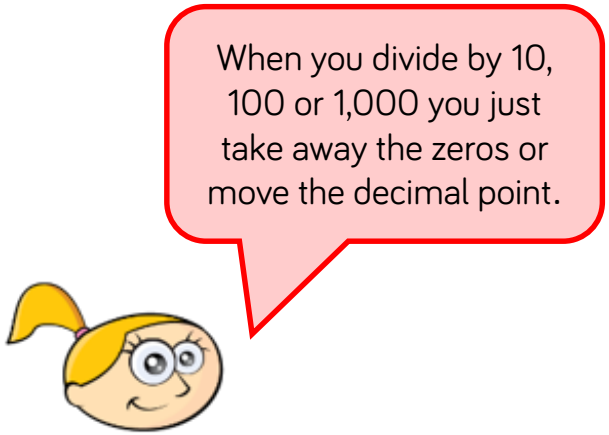
Can you find a path from 6 to 0.06?
You cannot make diagonal moves.

6	× 10	× 10	÷ 100
÷ 10	× 100	× 100	÷ 10
× 10	÷ 10	÷ 1,000	÷ 100
÷ 1,000	× 1,000	× 100	0.06

Is there more than one way?

6	× 10	× 10	÷ 100
÷ 10	× 100	× 100	÷ 10
× 10	÷ 10	÷ 1,000	÷ 100
÷ 1,000	× 1,000	× 100	0.06

Eva says,



Do you agree?
Explain why.

Eva is wrong, the decimal point never moves. When dividing, the digits move right along the place value columns.

Possible examples to prove Eva wrong:

24 ÷ 10 = 2.4

107 ÷ 10 = 17
This shows that you cannot just remove a zero from the number

Multiply Decimals by Integers

Notes and Guidance

Children use concrete resources to multiply decimals and explore what happens when you exchange with decimals.

Children use their skills in context and make links to money and measures.

Mathematical Talk

Which is bigger, 0.1, 0.01 or 0.001? Why?

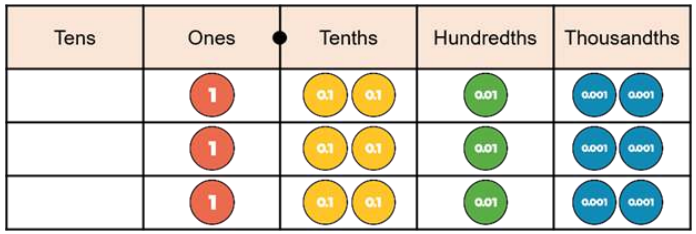
How many 0.1s do you need to exchange for a whole one?

Can you draw a bar model to represent the problem?

Can you think of another way to multiply by 5? (e.g. multiply by 10 and divide by 2).

Varied Fluency

- Use the place value counters to multiply 1.212 by 3
  Complete the calculation alongside the concrete representation.



- A jar of sweets weighs 1.213 kg.
  How much would 4 jars weigh?

- Rosie is saving her pocket money. Her mum says,

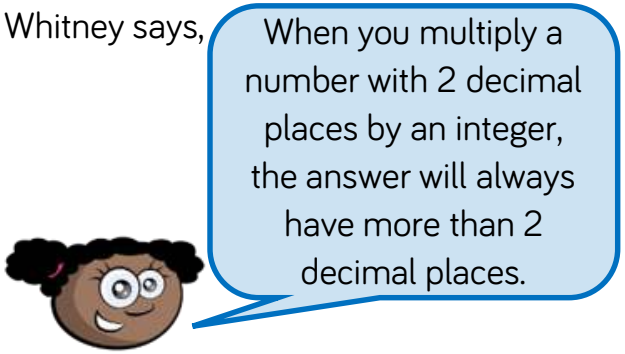
> "Whatever you save, I will give you five times the amount."

  If Rosie saves £2.23, how much will her mum give her?
  If Rosie saves £7.76, how much will her mum give her? How much will she have altogether?

Multiply Decimals by Integers

Reasoning and Problem Solving

Whitney says,



Do you agree?
Explain why.

Possible answer:

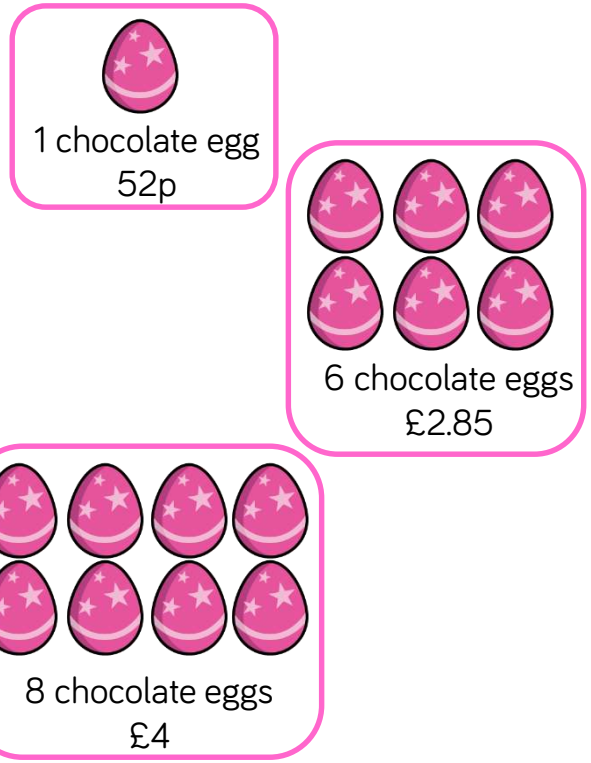
I do not agree because there are examples such as 2.23 × 2 that gives an answer with only two decimal places.

Fill in the blanks

	3	•	4	5
×				☐
	0	•	3	0
	☐	•	4	0
1	☐	•	0	0
	☐	☐	•	☐ ☐

	3	•	4	5
×				6
	0	•	3	0
	2	•	4	0
1	8	•	0	0
2	0	•	7	0

Chocolate eggs can be bought in packs of 1, 6 or 8
What is the cheapest way for Dexter to buy 25 chocolate eggs?



£11.92

He should buy four packs of 6 plus an individual egg.

Divide Decimals by Integers

Notes and Guidance

Children continue to use concrete resources to divide decimals and explore what happens when exchanges take place.

Children build on their prior knowledge of sharing and grouping when dividing and apply this skill in context.

Mathematical Talk

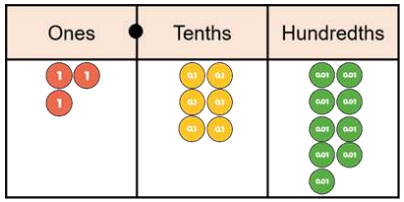
Are we grouping or sharing?

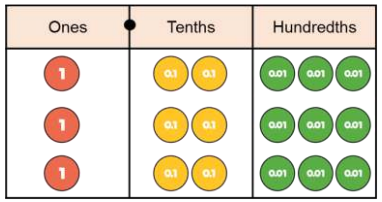
How else could we partition the number 3.69? (For example, 2 ones, 16 tenths and 9 hundredths.)

How could we check that our answer is correct?

Varied Fluency

- Divide 3.69 by 3
  Use the diagrams to show the difference between grouping and by sharing?





Use these methods to complete the sentences.
3 ones divided by 3 is _____ ones.
6 tenths divided by 3 is _____ tenths.
9 hundredths divided by 3 is _____ hundredths.
Therefore, 3.69 divided by 3 is _____

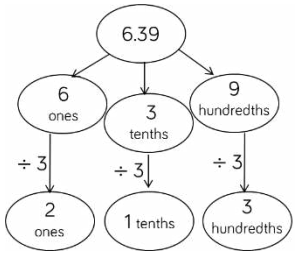
- Decide whether you will use grouping or sharing and use the place value chart and counters to solve:

  7.55 ÷ 5 8.16 ÷ 3 3.3 ÷ 6

- Amir solves 6.39 ÷ 3 using a part whole method.

  Use this method to solve

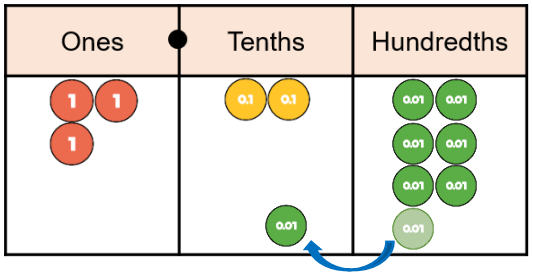
  8.48 ÷ 2 6.9 ÷ 3 6.12 ÷ 3



Divide Decimals by Integers

Reasoning and Problem Solving

When using the counters to answer 3.27 divided by 3, this is what Tommy did:



Tommy says,

I only had 2 counters in the tenths column, so I moved one of the hundredths so each column could be grouped in 3s.

Do you agree with what Tommy has done? Explain why.

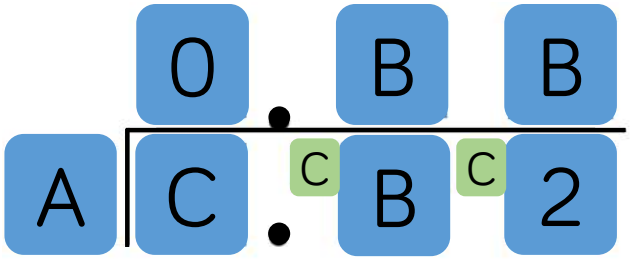
Possible answer:

Tommy is incorrect because he cannot move a hundredth to the tenths.
He should have exchanged the 2 tenths for hundredths to get an answer of 1.09

C is $\frac{1}{4}$ of A

B = C + 2

Use the clues to complete the division.



Children may try A as 8 and C as 2 but will realise that this cannot complete the whole division.

Therefore A is 4, B is 3 and C is 1

Division to Solve Problems

Notes and Guidance

Children will apply their understanding of division to solve problems in cases where the answer has up to 2 decimal places.

Children will continue to show division using place value counters and exchanging where needed.

Mathematical Talk

How can we represent this problem using a bar model?

How will we calculate what this item costs?

How will we use division to solve this?

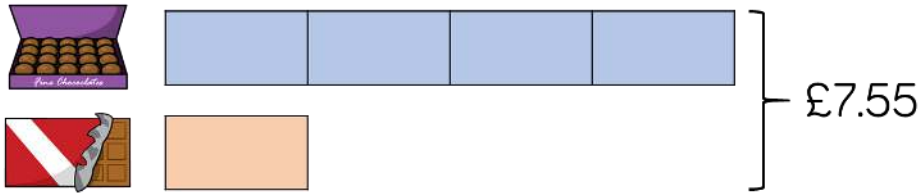
How will we label our bar model to represent this?

Varied Fluency

- Mrs Forbes has saved £4,960
  She shares the money between her 15 grandchildren.
  How much do they each receive?

- Modelling clay is sold in two different shops.
  Shop A sells four pots of clay for £7.68
  Shop B sells three pots of clay for £5.79
  Which shop has the better deal?
  Explain your answer.

- A box of chocolates costs 4 times as much as a chocolate bar.
  Together they cost £7.55



How much does each item cost?
How much more does the box of chocolates cost?

Division to Solve Problems

Reasoning and Problem Solving

Each division sentence can be completed using the digits below.

1 2 3 4 5 6

□ . 3 ÷ □ = 0.26

12 . □ ÷ □ = 4.2

4 . □ 8 ÷ □ = 1.07

1.3 ÷ 5 = 0.26
12.6 ÷ 3 = 4.2
4.28 ÷ 4 = 1.07

Jack and Rosie are both calculating the answer to 147 ÷ 4

Jack says,

The answer is 36 remainder 3

Rosie says,

The answer is 36.75

Who do you agree with?

They are both correct.

Rosie has divided her remainder of 3 by 4 to get 0.75 whereas Jack has recorded his as a remainder.

Decimals as Fractions

Notes and Guidance

Children explore the relationship between decimals and fractions. They start with a decimal and use their place value knowledge to help them convert it into a fraction.
Children will use their previous knowledge of exchanging between columns, for example, 3 tenths is the same as 30 hundredths.
Once children convert from a decimal to a fraction, they simplify the fraction to help to show patterns.

Mathematical Talk

How would you record your answer as a decimal and a fraction? Can you simplify your answer?

How would you convert the tenths to hundredths?

What do you notice about the numbers that can be simplified in the table?

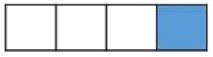
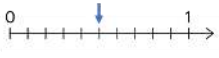
Can you have a unit fraction that is larger than 0.5? Why?

Varied Fluency

- What decimal is shaded?
  Can you write this as a fraction?

0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

- Complete the table.

Decimal	Fraction in tenths or hundredths	Simplified fraction
0.6	$\frac{6}{10}$	$\frac{3}{5}$
		
		
0.95		

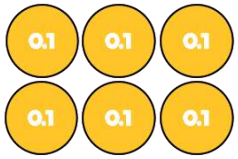
- Three friends share a pizza. Sam ate 0.25 of the pizza, Mark ate 0.3 of the pizza and Jill ate 0.35 of the pizza.
  - Can you write the amount each child ate as a fraction?
  - What fraction of the pizza is left?

Decimals as Fractions

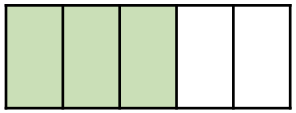
Reasoning and Problem Solving

Odd one out.

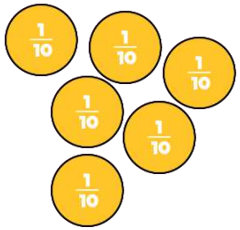
A



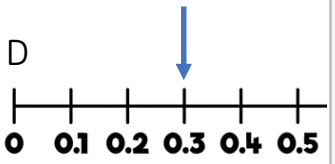
B



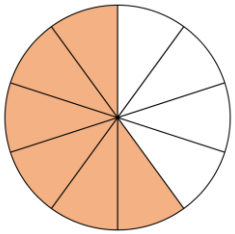
C



D



E



F

0.2×3

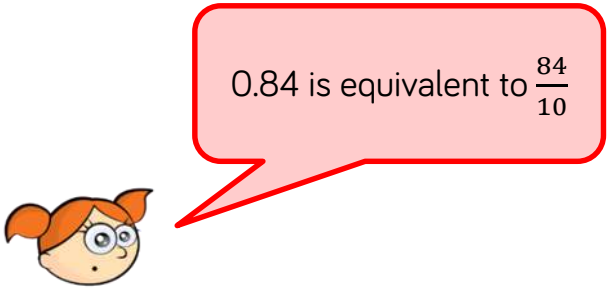
Which is the odd one out and why?

Possible response:

D is the odd one out because it shows 0.3

Explore how the rest represent 0.6

Alex says,



Do you agree?
Explain why.

Possible response:

Alex is wrong because 0.84 is 8 tenths and 4 hundredths and $\frac{84}{10}$ is 84 tenths.

Fractions to Decimals (1)

Notes and Guidance

At this point children should know common fractions, such as thirds, quarters, fifths and eighths, as decimals.

Children explore how finding an equivalent fraction where the denominator is 10, 100 or 1,000 makes it easier to convert from a fraction to a decimal.

They investigate efficient methods to convert fractions to decimals.

Mathematical Talk

How many hundredths are equivalent to one tenth?

How could you convert a fraction to a decimal?

Which is the most efficient method? Why?

Which equivalent fraction would be useful?

Varied Fluency

- Match the fractions to the equivalent decimals.

$\frac{2}{5}$	0.04
$\frac{1}{25}$	0.4
$\frac{1}{4}$	0.25

- Use your knowledge of known fractions to convert the fractions to decimals. Show your method for each one.

$\frac{7}{20}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{6}{200}$

- Mo says that $\frac{63}{100}$ is less than 0.65

Do you agree with Mo?
Explain your answer.

Fractions to Decimals (1)

Reasoning and Problem Solving

Amir says,

The decimal 0.42 can be read as 'four tenths and two hundredths'.

Teddy says,

The decimal 0.42 can be read as 'forty-two hundredths'.

Who do you agree with?
Explain your answer.

Both are correct. Four tenths are equivalent to forty hundredths, plus the two hundredths equals forty-two hundredths.

True or False?

0.3 is bigger than $\frac{1}{4}$

Explain your reasoning.

True because $\frac{1}{4}$ is 25 hundredths and 0.3 is 30 hundredths. Therefore, 0.3 is bigger.

Dora and Whitney are converting $\frac{30}{500}$ into a decimal.

- Dora doubles the numerator and denominator, then divides by 10
- Whitney divides both the numerator and the denominator by 5
- Both get the answer $\frac{6}{100}$ = 0.06

Which method would you use to work out each of the following?

$\frac{25}{500}$ $\frac{125}{500}$ $\frac{40}{500}$ $\frac{350}{500}$

Explain why you have used a certain method.

Possible response:

$\frac{25}{500}$ - divide by 5, known division fact.

$\frac{125}{500}$ - double, easier than dividing 125 by 5

$\frac{40}{500}$ - divide by 5, known division fact.

$\frac{350}{500}$ - double, easier than dividing 350 by 5

Fractions to Decimals (2)

Notes and Guidance

It is important that children recognise that $\frac{3}{4}$ is the same as $3 \div 4$. They can use this understanding to find fractions as decimals by then dividing the numerator by the denominator.

In the example provided, we cannot make any equal groups of 5 in the ones column so we have exchanged the 2 ones for 20 tenths. Then we can divide 20 into groups of 5

Mathematical Talk

Do we divide the numerator by the denominator or divide the denominator by the numerator? Explain why.

When do we need to exchange?

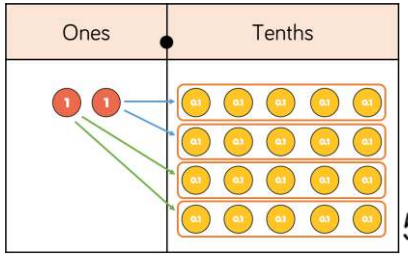
Are we grouping or are we sharing? Explain why.

Why is it useful to write 2 as 2.0 when dividing by 5?

Why is it not useful to write 5 as 5.0 when dividing by 8?

Varied Fluency

- Deena has used place value counters to write $\frac{2}{5}$ as a decimal. She has divided the numerator by the denominator.



$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \end{array}$$

Use this method to convert the fractions to decimals. Give your answers to 2 decimal places.

$\frac{1}{2}$ $\frac{3}{4}$

- Use the short division method to convert the fractions to decimals. Write the decimals to three decimal places.

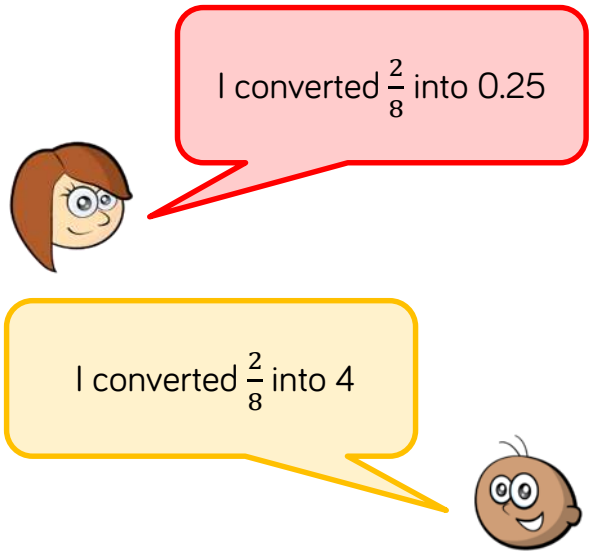
$\frac{5}{8}$ $\frac{4}{5}$ $\frac{8}{5}$

- 8 friends share 7 pizzas.
How much pizza does each person get?
Give your answer as a decimal and as a fraction.

Fractions to Decimals (2)

Reasoning and Problem Solving

Rosie and Tommy have both attempted to convert $\frac{2}{8}$ into a decimal.

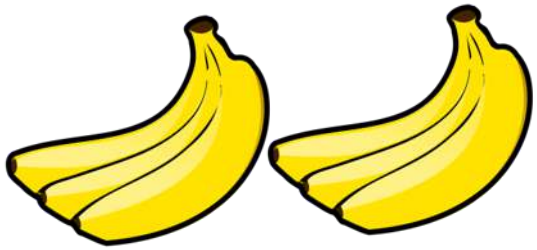


Who is correct?
Prove it.

Rosie is correct and Tommy is incorrect.

Tommy has divided 8 by 2 rather than 2 divided by 8 to find the answer.

Mo shares 6 bananas between some friends.



Each friend gets 0.75 of a banana.

How many friends does he share the bananas with?
Show your method.

Mo shares his 6 bananas between 8 friends because 6 divided by 8 equals 0.75

Children may show different methods:

Method 1: Children add 0.75 until they reach 6. This may involve spotting that 4 lots of 0.75 equals 3 and then they double this to find 8 lots of 0.75 equals 6

Method 2: Children use their knowledge that 0.75 is equivalent to $\frac{3}{4}$ to find the equivalent fraction of $\frac{6}{8}$

White Rose Maths

Spring - Block 2

Percentages

Overview

Small Steps

- Fractions to percentages
- Equivalent FDP
- Order FDP
- Percentage of an amount (1)
- Percentage of an amount (2)
- Percentages – missing values

NC Objectives

Solve problems involving the calculation of percentages [for example, of measures and such as 15% of 360] and the use of percentages for comparison.

Recall and use equivalences between simple fractions, decimals and percentages including in different contexts.

Fractions to Percentages

Notes and Guidance

It is important that children understand that 'percent' means 'out of 100'.
Children will be familiar with converting some common fractions from their work in Year 5
They learn to convert fractions to equivalent fractions where the denominator is 100 in order to find the percentage equivalent.

Mathematical Talk

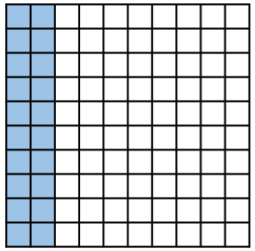
What does the word 'percent' mean?

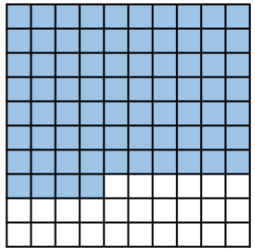
How can you convert tenths to hundredths?

Why is it easy to convert fiftieths to hundredths?
What other fractions are easy to convert to percentages?

Varied Fluency

- What fraction of each hundred square is shaded? Write the fractions as percentages.





- Complete the table.

Fraction	Percentage
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{10}$	
$\frac{1}{5}$	

- Fill in the missing numbers.

$\frac{12}{100} = \square\%$ $\qquad$ $\frac{\square}{100} = 35\%$

$\frac{12}{50} = \frac{\square}{100} = \square\%$ $\qquad$ $\frac{44}{\square} = \frac{22}{100} = 22\%$

Fractions to Percentages

Reasoning and Problem Solving

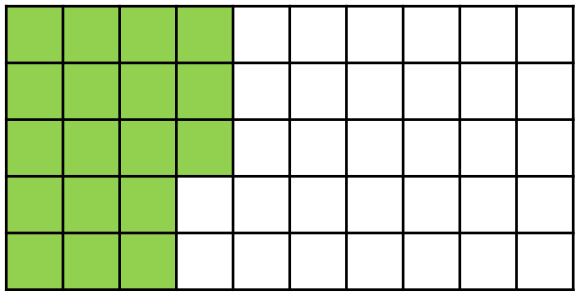
In a Maths test, Tommy answered 62% of the questions correctly.

Rosie answered $\frac{3}{5}$ of the questions correctly.

Who answered more questions correctly?

Explain your answer.

Tommy answered more questions correctly because $\frac{3}{5}$ as a percentage is 60% and this is less than 62%



Amir thinks that 18% of the grid has been shaded.

Dora thinks that 36% of the grid has been shaded.

Who do you agree with?

Explain your reasoning.

Dora is correct because $\frac{18}{50} = \frac{36}{100}$

Equivalent FDP

Notes and Guidance

Children use their knowledge of common equivalent fractions and decimals to find the equivalent percentage.

A common misconception is that 0.1 is equivalent to 1%. Diagrams may be useful to support understanding the difference between tenths and hundredths and their equivalent percentages.

Mathematical Talk

How does converting a decimal to a fraction help us to convert it to a percentage?

How do you convert a percentage to a decimal?

Can you use a hundred square to represent your conversions?

Varied Fluency

- Complete the table.

Decimal	Fraction	Percentage
0.35	$\frac{35}{100}$	35%
0.27		
0.6		
0.06		

- Use <, > or = to complete the statements.

0.36 ◯ 40% $\frac{7}{10}$ ◯ 0.07

0.4 ◯ 25% 0.4 ◯ $\frac{1}{4}$

- Which of these are equivalent to 60%?

$\frac{60}{100}$ $\frac{6}{100}$ 0.06 $\frac{3}{5}$ $\frac{3}{50}$ 0.6

Equivalent FDP

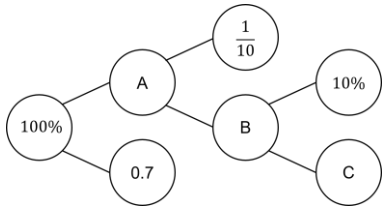
Reasoning and Problem Solving

Amir says 0.3 is less than 12% because 3 is less than 12

Explain why Amir is wrong.

Amir is wrong because 0.3 is equivalent to 30%

Complete the part-whole model.
How many different ways can you complete it?



Can you create your own version with different values?

A = 0.3, 30% or $\frac{3}{10}$

B = 0.2, 20%, $\frac{2}{10}$ or $\frac{1}{5}$

C = 0.1, 10% or $\frac{1}{10}$

How many different fractions can you make using the digit cards?

1 2 3 4 5 6

How many of the fractions can you convert into decimals and percentages?

Possible answers:

Children make a range of fractions.

They should be able to convert

$\frac{1}{2}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}$

and $\frac{4}{5}$ into decimals and percentages.

Order FDP

Notes and Guidance

Children convert between fractions, decimals and percentages to enable them to order and compare them.

Encourage them to convert each number to the same form so that they can be more easily ordered and compared. Once the children have compared the numbers, they will need to put them back into the original form to answer the question.

Mathematical Talk

What do you notice about the fractions, decimals or percentages? Can you compare any straight away?

What is the most efficient way to order them?

Do you prefer to convert your numbers to decimals, fractions or percentages? Why?

If you put them in ascending order, what will it look like?
If you put them in descending order, what will it look like?

Varied Fluency

- Use <, > or = to complete the statements:

60% ◯ 0.6 ◯ $\frac{3}{5}$

0.23 ◯ 24% ◯ $\frac{1}{4}$

37.6% ◯ $\frac{3}{8}$ ◯ 0.27

- Order from smallest to largest:

50% | $\frac{2}{5}$ | 0.45 | $\frac{3}{10}$ | 54% | 0.05

- Four friends share a pizza. Whitney eats 35% of the pizza, Teddy eats 0.4 of the pizza, Dora eats 12.5% of the pizza and Alex eats 0.125 of the pizza.

  Write the amount each child eats as a fraction.
  Who eats the most? Who eats the least? Is there any left?

Order FDP

Reasoning and Problem Solving

In his first Geography test, Mo scored 38%

In the next test he scored $\frac{16}{40}$

Did Mo improve his score?

Explain your answer.

Mo improved his score.

$\frac{16}{40}$ is equivalent to 40% which is greater than his previous score of 38%

Which month did Eva save the most money?

Estimate your answer using your knowledge of fractions, decimals and percentages.

Explain why you have chosen that month.

In January, Eva saves $\frac{3}{5}$ of her £20 pocket money.

In February, she saves 0.4 of her £10 pocket money.

In March, she saves 45% of her £40 pocket money.

She saved the most money in March.

Estimates:

Over £10 in January because $\frac{3}{5}$ is more than half.

Under £10 in February because she only had £10 to start with and 0.4 is less than half.

Nearly £20 in March because 45% is close to a half.

Percentage of an Amount (1)

Notes and Guidance

Children use known fractional equivalences to find percentages of amounts.
Bar models and other visual representations may be useful in supporting this e.g. 25% = $\frac{1}{4}$ so we divide into 4 equal parts.
In this step, we focus on 50%, 25%, 10% and 1% only.

Mathematical Talk

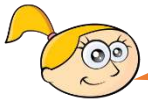
Why do we divide a quantity by 2 in order to find 50%?

How do you calculate 10% of a number mentally?

What's the same and what's different about 10% of 300 and 10% of 30?

Varied Fluency

- Eva says,



50% is equivalent to $\frac{1}{2}$
To find 50% of an amount, I can divide by 2

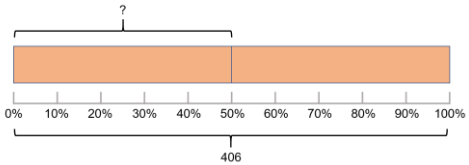
Complete the sentences.

25% is equivalent to $\frac{1}{\square}$ To find 25% of an amount, divide by ___

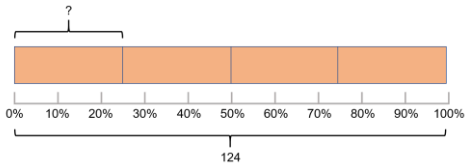
10% is equivalent to $\frac{1}{\square}$ To find 10% of an amount, divide by ___

1% is equivalent to $\frac{1}{\square}$ To find 1% of an amount, divide by ___

- Use the bar models to help you complete the calculations.



50% of 406 =



25% of 124 =

- Find:

50% of 300	25% of 300	10% of 300	1% of 300
50% of 30	25% of 30	10% of 30	1% of 30
50% of 60	25% of 60	10% of 60	1% of 60

Percentage of an Amount (1)

Reasoning and Problem Solving

Mo says,

To find 10% you divide by 10, so to find 50% you divide by 50

Do you agree? Explain why.

Possible answer:

Mo is wrong because 50% is equivalent to a half so to find 50% you divide by 2

Eva says to find 1% of a number, you divide by 100
Whitney says to find 1% of a number, you divide by 10 and then by 10 again.

Who do you agree with?
Explain your answer.

They are both correct.
Whitney has divided by 100 in two smaller steps.

Complete the missing numbers.

50% of 40 = ___% of 80

___% of 40 = 1% of 400

10% of 500 = ___% of 100

25

10

50

Percentage of an Amount (2)

Notes and Guidance

Children build on the last step by finding multiples of 10% and other known percentages.
They explore different methods of finding certain percentages e.g. Finding 20% by dividing by 10 and multiplying by 2 or by dividing by 5. They also explore finding 5% by finding half of 10%. Using these methods, children build up to find percentages such as 35%.

Mathematical Talk

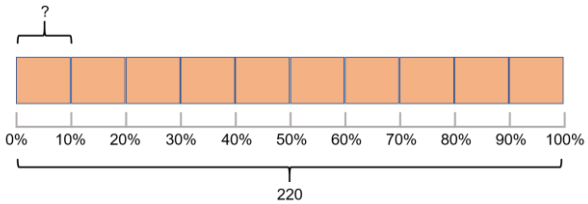
Is dividing by 10 and multiplying by 5 the most efficient way to find 50%? Explain why.

Is dividing by 10 and multiplying by 9 the most efficient way to find 90%? Explain why.

How many ways can you think of to calculate 60% of a number?

Varied Fluency

- Mo uses a bar model to find 30% of 220



10% of 220 = 22 , so 30% of 220 = 3 × 22 = 66

Use Mo's method to calculate:

40% of 220 20% of 110 30% of 440 90% of 460

- To find 5% of a number, divide by 10 and then divide by 2
Use this method to work out:
(a) 5% of 140 (b) 5% of 260 (c) 5% of 1 m 80 cm

How else could we work out 5%?

- Calculate:

15% of 60 m 35% of 300 g 65% of £20

Percentage of an Amount (2)

Reasoning and Problem Solving

Four children in a class were asked to find 20% of an amount, this is what they did:

Whitney: I divided by 5 because 20% is the same as one fifth

Amir: I found one percent by dividing by 100, then I multiplied my answer by 20

Alex: I did 10% add 10%

Jack: I found ten percent by dividing by 10, then I multiplied my answer by 2

Who do you think has the most efficient method? Explain why.
Who do you think will end up getting the answer incorrect?

All methods are acceptable ways of finding 20% Children may have different answers because they may find different methods easier. Discussion could be had around whether or not their preferred method is always the most efficient.

How many ways can you find 45% of 60?

Use similar strategies to find 60% of 45

What do you notice?

Does this always happen?
Can you find more examples?

Possible methods include:

10% × 4 + 5%

25% + 20%

25% + 10%+ 10%

50% – 5%

To find 60% of 45

10% × 6

50% + 10%

10% × 3

Children will notice that 45% of 60 = 60% of 45

This always happens.

Percentages – Missing Values

Notes and Guidance

Children use their understanding of percentages to find the missing whole or a missing percentage when the other values are given. They may find it useful to draw a bar model to help them see the relationship between the given percentage or amount and the whole.

It is important that children see that there may be more than one way to solve a problem and that some methods are more efficient than others.

Mathematical Talk

If we know a percentage, can we work out the whole?

If we know the whole and the amount, can we find what percentage has been calculated?

What diagrams could help you visualise this problem?
Is there more than one way to solve the problem?

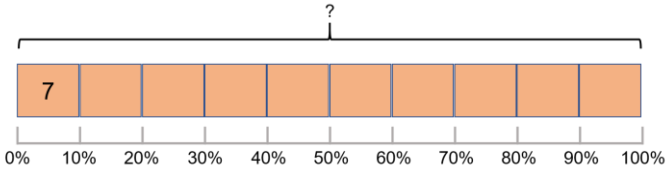
What is the most efficient way to find a missing value?

Varied Fluency

- 350,000 people visited the Natural History Museum last week.
  15% of the people visited on Monday.
  40% of the people visited on Saturday.
  How many people visited the Natural History Museum during the rest of the week?

- If 7 is 10% of a number, what is the number?

  Use the bar model to help you.



- Complete:

10% of 150 = ☐ 30% of ☐ = 45

30% of 300 = ☐ 30% of ☐ = 900

Can you see a link between the questions?

Percentages – Missing Values

Reasoning and Problem Solving

What percentage questions can you ask about this bar model?

Possible answer:
If 20% of a number is 3.5, what is the whole?
What is 60%?
What is 10%?

Fill in the missing values to make this statement correct.
Can you find more than one way?

25% of ☐ = ☐ % of 60

Possible answers:

25% of 60 = 25% of 60

25% of 120 = 50% of 60

25% of 24 = 10% of 60

25% of 2.4 = 1% of 60

25% of 180 = 75% of 60

A golf club has 200 members.

58% of the members are male.
50% of the female members are children.

(a) How many male members are in the golf club?
(b) How many female children are in the golf club?

.

116 male members

42 female children

White Rose Maths

Spring - Block 3

Algebra

Overview

Small Steps

- Find a rule – one step
- Find a rule – two step
- Forming expressions
- Substitution
- Formulae
- Forming equations
- Solve simple one-step equations
- Solve two-step equations
- Find pairs of values
- Enumerate possibilities

NC Objectives

Use simple formulae.

Generate and describe linear number sequences.

Express missing number problems algebraically.

Find pairs of numbers that satisfy an equation with two unknowns.

Enumerate possibilities of combinations of two variables.

Find a Rule – One Step

Notes and Guidance

Children explore simple one-step function machines. Explain that a one-step function is where they perform just one operation on the input.
Children understand that for each number they put into a function machine, there is an output. They should also be taught to "work backwards" to find the input given the output. Given a set of inputs and outputs, they should be able to work out the function.

Mathematical Talk

What do you think "one-step function" means?
What examples of functions do you know?
Do some functions have more than one name?
What do you think input and output mean?
What is the output if ….?
What is the input if ….?
How many sets of inputs and outputs do you need to be able to work out the function? Explain how you know.

Varied Fluency

- Here is a function machine.

Input → × 4 → Output

- What is the output if the input is 2?
- What is the output if the input is 7.2?
- What is the input if the output was 20?
- What is the input if the output was 22?

- Complete the table for the function machine.

Input → + 5 → Output

Input	5	5.8	10	– 3	– 8			
Output						9	169	0

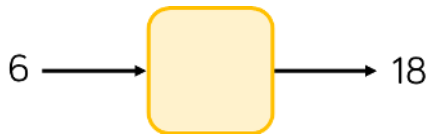
- Find the missing function.

10 → 5
24 → 12
7 → 3.5

Find a Rule – One Step

Reasoning and Problem Solving

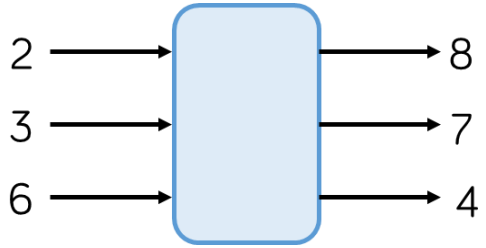
Eva has a one-step function machine. She puts in the number 6 and the number 18 comes out.



What could the function be?
How many different answers can you find?

The function could be + 12, × 3

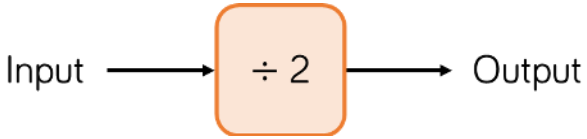
Amir puts some numbers into a function machine.



What is the output from the function when the input is 16?

The function is subtract from 10 so the output is −6

Dora puts a number into the function machine.



Dora's number is:
- A factor of 32
- A multiple of 8
- A square number

What is Dora's input?
What is her output?

Can you create your own clues for the numbers you put into a function machine for a partner to solve?

Dora's input is 16

Her output is 8

Find a Rule – Two Step

Notes and Guidance

Children build on their knowledge of one-step functions to look at two-step function machines. Discuss with children whether a function such as + 5 and + 6 is a two-step function machine or whether it can be written as a one-step function.
Children look at strategies to find the functions. They can use trial and improvement or consider the pattern of differences.
Children record their input and output values in the form of a table.

Mathematical Talk

How can you write + 5 followed by – 2 as a one-step function?
If I change the order of the functions, is the output the same?
What is the output if ….?
What is the input if ….?
If you add 3 to a number and then add 5 to the result, how much have you added on altogether?

Varied Fluency

- Here is a function machine.

Input → × 2 → + 5 → Output

  - What is the output if the input is 5?
  - What is the input if the output is 19?
  - What is the input if the output is 3.5?

- Complete the table for the given function machine.

Input → × 3 → – 4 → Output

Input	1	2	3	4	5
Output					

  - What patterns do you notice in the outputs?
  - What is the input if 20 is the output? How did you work it out?

- How can you write this two-step machine as a one-step machine?

Input → + 3 → + 5 → Output

Check your answer by inputting values.

Find a Rule – Two Step

Reasoning and Problem Solving

Teddy has two function machines.

Input → + 5 → × 2 → Output

Input → × 2 → + 5 → Output

He says,

The function machines will give the same answer.

Is Teddy correct?

Is there an input that will give the same output for both machines?

No they do not give the same answer. Encourage children to refer to the order of operations to help them understand why the outputs are different.

Mo has the following function machines.

Input → + 2 → – 8 → Output

Input → × 8 → ÷ 2 → Output

Input → × 2 → – 8 → Output

Explain which of these can be written as single function machines.

The first one can be written as – 6

The second can be written as × 4

The third cannot be written as a single machine.

Forming Expressions

Notes and Guidance

Children have now met one-step and two-step function machines with numerical inputs.
In this step, children use simple algebraic inputs e.g. y.
Using these inputs in a function machine leads them to forming expressions e.g. $y + 4$. The use of cubes to represent a variable can aid understanding.
Children are introduced to conventions that we use when writing algebraic expressions. e.g. $y \times 4$ as $4y$.

Mathematical Talk

What expressions can be formed from this function machine?

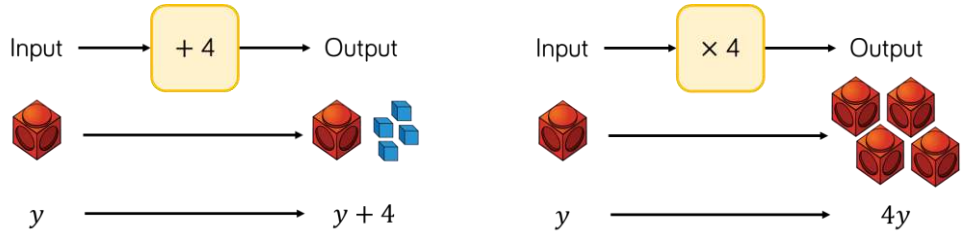
What would the function machine look like for this rule/expression?

How can you write $x \times 3 + 6$ differently?

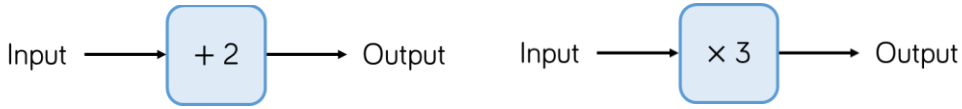
Are $2a + 6$ and $6 + 2a$ the same? Explain your answer

Varied Fluency

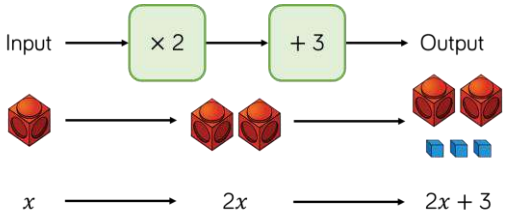
- Mo uses cubes to write expressions for function machines.



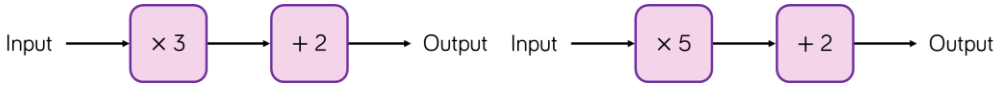
Use Mo's method to represent the function machines.
What is the output for each machine when the input is a?



- Eva is writing expressions for two-step function machines.



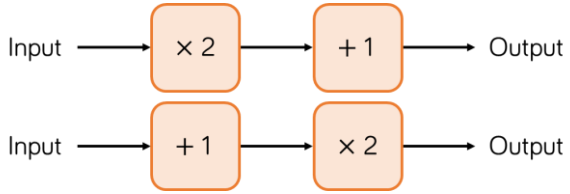
Use Eva's method to write expressions for the function machines.



Forming Expressions

Reasoning and Problem Solving

Amir inputs m into these function machines.



He says the outputs of the machines will be the same.

Do you agree?

Explain your answer.

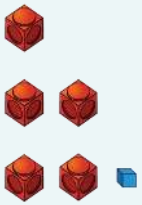
No, because $2m + 1$ isn't the same as $2m + 2$

$\underline{2m + 1}$

Input

× 2

+ 1



$\underline{2m + 2}$

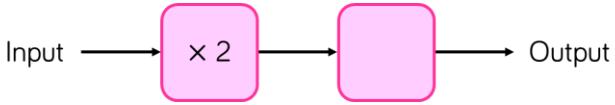
Input

+ 1

× 2

Children may use examples with numbers to show this.

This function machine gives the same output for every input.
For example if the input is 5 then the output is 5 and so on.



What is the missing part of the function?

What other pairs of functions can you think that will do the same?

÷ 2

Other pairs of functions that will do the same are functions that are the inverse of each other e.g. + 3, – 3

Substitution

Notes and Guidance

Children substitute into simple expressions to find a particular value.

They have already experienced inputting into a function machine, and teachers can make the links between these two concepts.

Children will need to understand that the same expression can have different values depending on what has been substituted.

Mathematical Talk

Which letter represents the star?
Which letter represents the heart?
Would it still be correct if it was written as $a + b + c$?

What does it mean when a number is next to a letter?

Is $a + b + b$ the same as $a + 2b$?

Varied Fluency

- If ★ = 7 and ♥ = 5, what is the value of:

  ★ + ♥ + ♥

  If $a = 7$ and $b = 5$ what is the value of:

  $$a + b + b$$

  What is the same and what is different about this question?

- Substitute the following to work out the values of the expressions.

  $w = 3 \quad x = 5 \quad y = 2.5$

  - $w + 10$
  - $w + x$
  - $y - w$

- Substitute the following to work out the values of the expressions.

  $w = 10 \quad x = \frac{1}{4} \quad y = 2.5$

  - $3y$
  - wx
  - $12 + 8.8w$
  - $wy + 4x$

Substitution

Reasoning and Problem Solving

Here are two formulae.

$$p = 2a + 5$$

$$c = 10 - p$$

Find the value of c when $a = 10$

$c = -15$

$$x = 2c + 6$$

Whitney says,

$x = 12$ because c must be equal to 3 because it's the 3rd letter in the alphabet

Is Whitney correct?

No Whitney is incorrect. c could have any value.

Amir says,

When $c = 5$, $x = 31$

Amir is wrong.
Explain why.
What would the correct value of x be?

Amir has put the 2 next to the 5 to make 25 instead of multiplying 2 by 5

The correct value of x would be 16

Formulae

Notes and Guidance

Children substitute into familiar formulae such as those for area and volume.

They also use simple formulae to work out values of everyday activities such as the cost of a taxi or the amount of medicine to take given a person's age.

Mathematical Talk

What tells you something is a formula?

Which of the rectangles is the odd one out? Why?

Could you write the formula for a rectangle in a different way?

What other formulae do you know?

Varied Fluency

- Which of the following is a formula?

$P = 2l + 2w$ | $3d + 5$ | $20 = 3x - 2$

Explain why the other two are not formulae.

- Eva uses the formula $P = 2l + 2w$ to find the perimeter of rectangles.
Use this formula to find the perimeter of rectangles with the following lengths and widths.
  - $l = 15, w = 4$
  - $l = \frac{1}{4}, w = \frac{3}{8}$
  - $l = w = 5.1$

- This is the formula to work out the cost of a taxi.

$$C = 1.50 + 0.3m$$

C = the cost of the journey in £
m = number of miles travelled.
Work out the cost of a 12-mile taxi journey

Formulae

Reasoning and Problem Solving

Jack and Dora are using the following formula to work out what they should charge for four hours of cleaning.

Cost in pounds = 20 + 10 × number of hours

Jack thinks they should charge £60
Dora thinks they should charge £120

Who do you agree with?
Why?

Jack is correct as multiplication should be performed first following the order of operations.

Dora has not used the order of operations – she has added 20 and 10 and then multiplied 30 by 4

The rule for making scones is use 4 times as much flour (f) as butter (b).

Which is the correct formula to represent this?

A

$$f = \frac{b}{4}$$

B

$$f = 4b$$

C

$$f = b + 4$$

D

$$4f = b$$

Explain why the others are incorrect.

B is correct.

A shows the amount of flour is a quarter of the amount of butter.

C shows the amount of flour is 4 more than butter.

D shows butter is 4 times the amount of flour.

Forming Equations

Notes and Guidance

Building on the earlier step of forming expressions, children now use algebraic notation to form one-step equations. They need to know the difference between an expression like $x + 5$, which can take different values depending on the value of x, and an equation like $x + 5 = 11.2$ where x is a specific unknown value. This is best introduced using concrete materials e.g. cubes, can be used to represent the unknown values with counters being used to represent known numbers.

Mathematical Talk

What does the cube represent?
What do the counters represent?

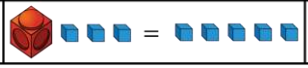
Design your own 'think of a number' problems.

What's the difference between an expression and an equation?

What's the difference between a formula and an equation?

Varied Fluency

- Amir represents a word problem using cubes, counters and algebra.

Words	Concrete	Algebra
I think of a number		x
Add 3		$x + 3$
My answer is 5		$x + 3 = 5$

Complete this table using Amir's method.

Words	Concrete	Algebra
I think of a number		
Add 1		
My answer is 8		

- A book costs £5 and a magazine costs £n
  The total cost of the book and magazine is £8
  Write this information as an equation.

- Write down algebraic equations for these word problems.
  - I think of a number, subtract 17, my answer is 20
  - I think of a number, multiply it by 5, my answer is 45

Forming Equations

Reasoning and Problem Solving

Rosie thinks of a number. She adds 7 and divides her answer by 2

Teddy thinks of a number. He multiples by 3 and subtracts 4

Rosie and Teddy think of the same number.
Rosie's answer is 9
What is Teddy's answer?

They both think of 11, therefore Teddy's answer is 29

Rosie and Teddy think of the same number again. This time, they both get the same answer.

Use trial and improvement to find the number they were thinking of.

They think of 3 and the answer they both get is 5

Eva spends 92p on yo-yos and sweets

She buys y yo-yos costing 11p and s sweets costing 4p.

Can you write an equation to represent what Eva has bought?

How many yo-yos and sweets could Eva have bought?

$92 = 11y + 4s$

She could have bought 1 sweet and 8 yo-yos or 4 yo-yos and 12 sweets.

Can you write a similar word problem to describe this equation?

$$74 = 15t + 2m$$

One-step Equations

Notes and Guidance

Children solve simple one step equations involving the four operations.

Children should explore this through the use of concrete materials such as cubes, counters and cups.

It is recommended that children learn to solve equations using a balancing method using inverse operations.

Mathematical Talk

Can you make some of your own equations using cups and counters for a friend to solve?

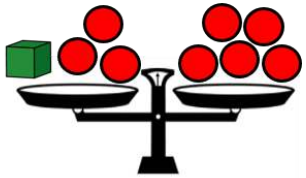
Why do you think the equation is set up on a balance? What does the balance represent? How does this help you solve the equation?

What is the same and what is different about each bar model?

Varied Fluency

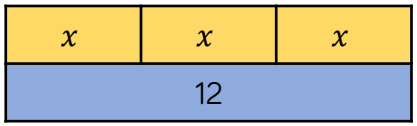
- How many counters is each cup worth?
  Write down and solve the equation represented by the diagram.

- Solve the equation represented on the scales.
  Can you draw a diagram to go with the next step?

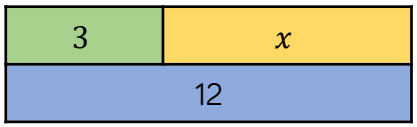


- Match each equation to the correct bar model and then solve to find the value of x.

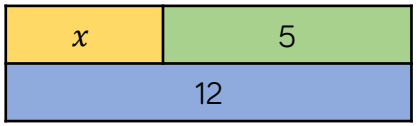
$x + 5 = 12$



$3x = 12$



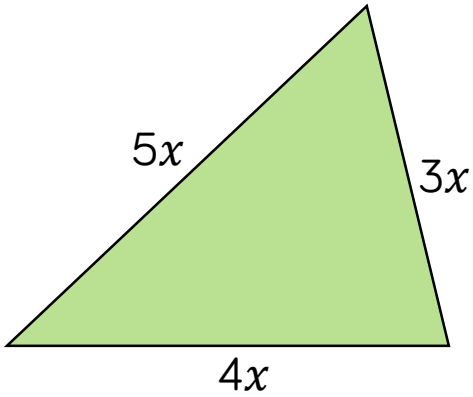
$12 = 3 + x$



One-step Equations

Reasoning and Problem Solving

The perimeter of the triangle is 216 cm.



Form an equation to show this information.

Solve the equation to find the value of x.

Work out the lengths of the sides of the triangle.

$3x + 4x + 5x = 216$

$12x = 216$

$x = 18$

$5 \times 18 = 90$

$3 \times 18 = 54$

$4 \times 18 = 72$

- Hannah is 8 years old
- Jack is 13 years old
- Grandma is $x + 12$ years old.
- The sum of their ages is 100

Form and solve an equation to work out how old Grandma is.

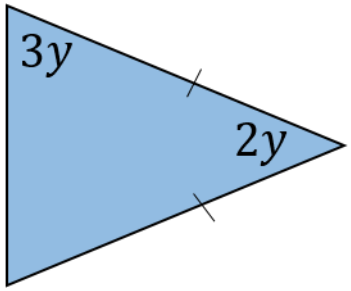
$8 + 13 + x + 12 = 100$

$33 + x = 100$

$x = 77$

Grandma is 77 years old.

What is the size of the smallest angle in this isosceles triangle?



How can you check your answer?

.

$8y = 180$

$y = 22.5$

Smallest angle = 45°

Check by working them all out and see if they add to 180°

Two-step Equations

Notes and Guidance

Children progress from solving equations that require one-step to equations that require two steps.
Children should think of each equation as a balance and solve it through doing the same thing to each side of the equation.
This should be introduced using concrete and pictorial methods alongside the abstract notation as shown. Only when secure in their understanding should children try this without the support of bar models or similar representations.

Mathematical Talk

Why do you have to do the same to each side of the equation?

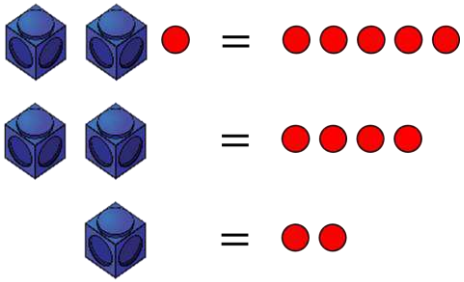
Why subtract 1? What does this do to the left hand side of the equation?

Does the order the equation is written in matter?

What's the same and what's different about solving the equations $2x + 1 = 17$ and $2x - 1 = 17$?

Varied Fluency

Here is each step of an equation represented with concrete resources.



$2x + 1 = 5$

$-1 \qquad -1$

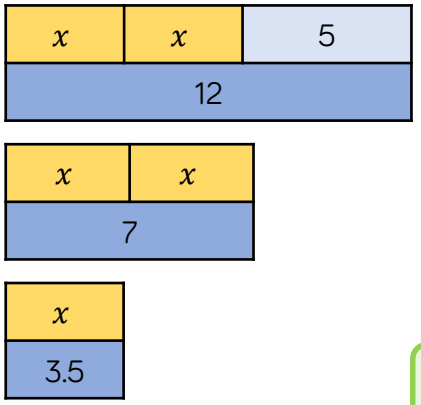
$2x = 4$

$\div 2 \qquad \div 2$

$x = 2$

Use this method to solve:

$4y + 2 = 6 \qquad 9 = 2x + 5 \qquad 1 + 5a = 16$



Here is each step of an equation represented by a bar model. Write the algebraic steps that show the solution of the equation.

Use bar models to solve these equations.

$3b + 4 = 19 \qquad 20 = 4b + 2$

Two-step Equations

Reasoning and Problem Solving

The length of a rectangle is $2x + 3$
The width of the same rectangle is $x - 2$
The perimeter is 17 cm.

Find the area of the rectangle.

$6x + 2 = 17$
$6x = 15$
$x = 2.5$
Length = 8 cm
Width = 0.5 cm
Area = 4 cm^2

Alex has some algebra expression cards.

$y + 4$

$2y$

$3y - 1$

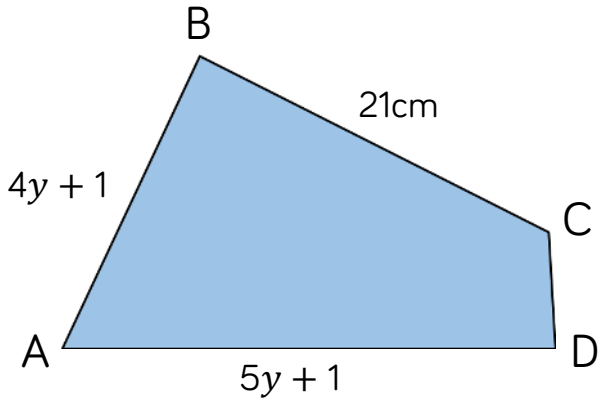
The mean of the cards is 19
Work out the value of each card.

$6y + 3 = 57$
$6y = 54$
$y = 9$

Card values:
13
18
26

Here is the quadrilateral ABCD.

The perimeter of the quadrilateral is 80 cm.



AB is the same length as BC.

Find the length of CD.

$4y + 1 = 21$
$4y = 20$
$y = 5$

AB = 21 cm
BC = 21 cm
AD = 26 cm
CD = 80 – (21 + 21 + 26) = 12 cm

Find Pairs of Values (1)

Notes and Guidance

Children use their understanding of substitution to consider what possible values a pair of variables can take.

At this stage we should focus on integer values, but other solutions could be a point for discussion.

Children can find values by trial and improvement, but should be encouraged to work systematically.

Mathematical Talk

Can a and b be the same value?

Is it possible for a or b to be zero?

How many possible integer answers are there? Convince me you have them all.

What do you notice about the values of c and d?

Varied Fluency

- a and b are variables:

$$a + b = 6$$

There are lots of possible solutions to This equation.
Find 5 different possible integer values for a and b.

a	b

- X and Y are whole numbers.
  - X is a one digit odd number.
  - Y is a two digit even number.
  - $X + Y = 25$

  Find all the possible pairs of numbers that satisfy the equation.

- $$c \times d = 48$$

  What are the possible integer values of c and d?
  How many different pairs of values can you find?

Find Pairs of Values (1)

Reasoning and Problem Solving

a, b and c are integers between 0 and 5

$$a + b = 6$$
$$b + c = 4$$

Find the values of a, b and c

How many different possibilities can you find?

Possible answers:

$a = 4 \quad b = 2$
$c = 2$

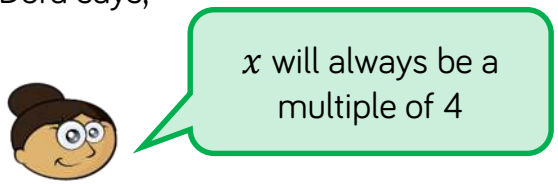
$a = 3 \quad b = 3$
$c = 1$

$a = 2 \quad b = 4$
$c = 0$

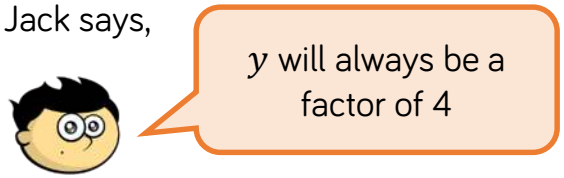
x and y are both positive whole numbers.

$$\frac{x}{y} = 4$$

Dora says,



Jack says,



Only one is correct – who is it?
Explain your answer.

Possible answer:

Dora is correct as x will always have to divide into 4 equal parts e.g. $32 \div 8 = 4$, $16 \div 4 = 4$

Jack is incorrect. $40 \div 10 = 4$ and 10 is not a factor of 4

Find Pairs of Values (2)

Notes and Guidance

Building on from the last step, children find possible solutions to equations which involve multiples of one or more unknown.

They should be encouraged to try one number for one of the variables first and then work out the corresponding value of the other variable. Children should then work systematically to test if there are other possible solutions that meet the given conditions.

Mathematical Talk

What does $2a$ mean? (2 multiplied by an unknown number)
What is the greatest/smallest number 'a' can be?

What strategy did you use to find the value of 'b'?

Can you draw a bar model to represent the following equations:
$3f + g = 20$
$7a + 3b = 40$
What could the letters represent?

Varied Fluency

- In this equation, a and b are both whole numbers which are less than 12.

$$2a = b$$

Write the calculations that would show all the possible values for a and b.

- Chose values of x and use the equation to work out the values of y.

$$7x + 4 = y$$

Value of x	Value of y

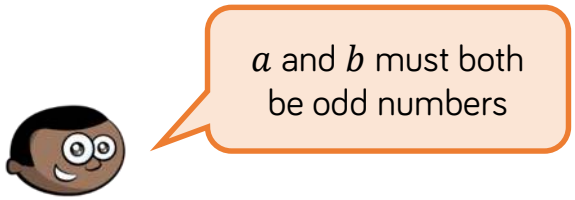
- $2g + w = 15$
g and w are positive whole numbers.
Write down all the possible values for g and w, show each of them in a bar model.

Find Pairs of Values (2)

Reasoning and Problem Solving

$ab + b = 18$

Mo says,



Is Mo correct?
Explain your answer.

Possible answer:

Mo is incorrect. Children may give examples to prove Mo is correct e.g. if $a = 5$ and $b = 3$, but there are also examples to show he is incorrect e.g. $a = 2$ and $b = 6$ where a and b are both even.

Large beads cost 5p and small beads cost 4p

Rosie has 79p to spend on beads.

4p 5p

How many different combinations of small and large beads can Rosie buy?

Can you write expressions that show all the solutions?

Possible answers:

$3l + 16s$
$7l + 11s$
$11l + 6s$
$15l + s$

White Rose Maths

Spring - Block 4

Converting Units

Overview

Small Steps

- Metric measures
- Convert metric measures
- Calculate with metric measures
- Miles and kilometres
- Imperial measures

NC Objectives

Solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate.

Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to 3 dp.

Convert between miles and kilometres.

Metric Measures

Notes and Guidance

Children read, write and recognise all metric measures for length, mass and capacity. They may need to be reminded the difference between capacity (the amount an object can contain) and volume (the amount actually in an object).

They develop their estimation skills in context and decide when it is appropriate to use different metric units of measure.

Mathematical Talk

Which units measure length? Mass? Capacity?

When would you use km instead of m? When would you use mm instead of cm?

Which is the most appropriate unit to use to measure the object? Explain your answer.

Why do you think _____ is not an appropriate estimate?

Varied Fluency

- Choose the unit of measure that would be the most appropriate to measure the items.

cm kg km g tonnes ml mm litres

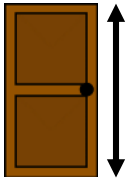
- The weight of an elephant
- The volume of water in a bath
- The length of an ant
- The length of a football pitch
- The weight of an apple

- Estimate how much juice the glass holds:

250 ml 2 litres 0.5 litres $\frac{1}{2}$ kg

- Estimate the height of the door frame:

20 mm 20 cm 20 m 2 km 2 m 0.2 km

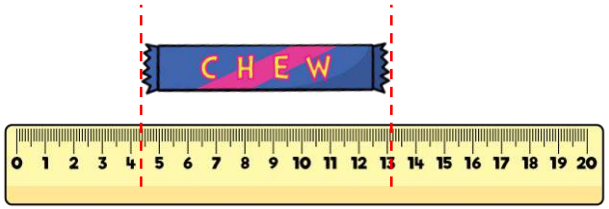


Metric Measures

Reasoning and Problem Solving

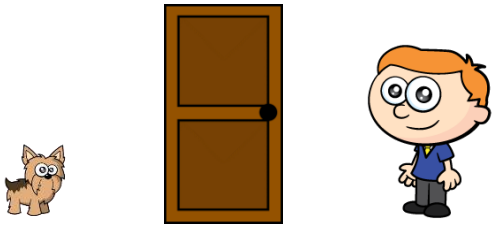
Teddy thinks his chew bar is 13.2 cm long.

Do you agree? Explain why.



Teddy is wrong because he has not lined up the end of his chew bar with zero.
It is actually 8.8 cm long.

Ron's dog is about $\frac{1}{4}$ of the height of the door.
Ron is three times the height of his dog.
Estimate the height of Ron and his dog.



Door = 2 m (200 cm)
Dog = 50 cm
Ron = 150 cm

Here is a train timetable showing the times of trains travelling from Halifax to Leeds.

Halifax	Leeds
07:33	08:09
07:49	08:37
07:52	08:51

An announcement states all trains will arrive $\frac{3}{4}$ of an hour late.
Which train will arrive in Leeds closest to 09:07?

The first train from Halifax, which will now arrive in Leeds at 08:54.

Convert Metric Measures

Notes and Guidance

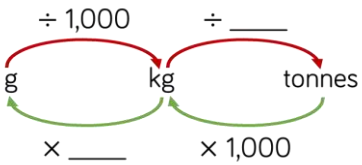
Children will use their skills of multiplying and dividing by 10, 100 and 1,000 when converting between units of length, mass and capacity.
Children will convert in both directions e.g. m to cm and cm to m. Using metre sticks and other scales will support this step.
They will need to understand the role of zero as a place holder when performing some calculations, as questions will involve varied numbers of decimal places.

Mathematical Talk

How could you work out what each mark is worth on the scales?
What do you think would be the most efficient method for converting the units of time?
What's the same and what's different between 1.5 km and 1.500 km? Are the zeroes needed? Why or why not?
What do you notice about the amounts in the table? Can you spot a pattern?
What's the same and what's different about km and kg?

Varied Fluency

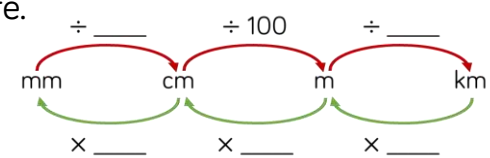
- There are ___ grams in one kilogram.
  There are ___ kilograms in one tonne.
  Use these facts to complete the tables.



g	kg
1,500	
	2.05
1,005	

kg	tonnes
1,202	
	4.004
125	

- There are ___ mm in one centimetre.
  There are ___ cm in one metre.
  There are ___ m in one kilometre.
  Use these facts to complete the table.



mm	cm	m	km
44,000			
	2,780		
		15.5	
			1.75

Convert Metric Measures

Reasoning and Problem Solving

Mo thinks that 12,000 g is greater than 20 kg because 12,000 > 20

Explain why Mo is wrong.

12,000 g = 12 kg, which is less than 20 kg.

Put these capacities in order, starting with the smallest.

3 litres	3,500 ml
0.4 litres	0.035 litres
450 ml	330 ml

0.035 litres
330 ml
0.4 litres
450 ml
3 litres
3,500 ml

A shop sells one-litre bottles of water for 99p each.

300 ml bottles of water are on offer at 8 bottles for £2

Whitney wants to buy 12 litres of water. Find the cheapest way she can do this.

£11.88 to buy 12 one-litre bottles.

12 litres = 40 bottles of size 300 ml.
40 ÷ 8 = 5 so this will cost
5 × 2 = £10
Whitney should buy 40 bottles of 300 ml.

Calculate with Metric Measures

Notes and Guidance

Children use and apply their conversion skills to solve measurement problems in context.

Teachers should model the use of pictorial representations, such as bar models, to represent the problem and help them decide which operation to use.

Mathematical Talk

What operation are you going to use and why?

How could you use a bar model to help you understand the question?

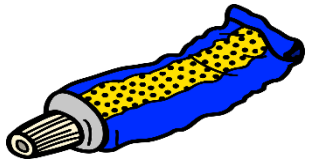
How many ___ are there in a ___?

How can we convert between ___ and ___?

Varied Fluency

- A tube of toothpaste holds 75 ml.

  How many tubes can be filled using 3 litres of toothpaste?



- A parcel weighs 439 grams. How much would 27 parcels weigh? Give your answer in kilograms.

- To bake buns for a party, Ron used these ingredients:

600 g caster sugar
0.6 kg butter
18 eggs (792 g)
$\frac{3}{4}$ kg self-raising flour
10 g baking powder

What is the total mass of the ingredients?
Give your answer in kilograms.

Calculate with Metric Measures

Reasoning and Problem Solving

Jack, Alex and Amir jumped a total of 12.69 m in a long jump competition. Alex jumped exactly 200 cm further than Jack. Amir jumped exactly 2,000 mm further than Alex. What distance did they all jump? Give your answers in metres.	Jack jumped 2.23 m. Alex jumped 4.23 m. Amir jumped 6.23 m.
Dora made a stack of her magazines. Each magazine on the pile is 2.5 mm thick. The total height of the stack is 11.5 cm high. How many magazines does she have in her pile?	There are 46 magazines in Dora's pile.

Each nail weighs 3.85 grams. There are 24 nails in a packet. What would be the total mass of 60 packets of nails? Give your answer in kilograms. How many packets would you need if you wanted $\frac{1}{2}$ kg of nails? How many grams of nails would be left over?	5.544 kg 6 packets (554.4 g) 55.4 g left over

Miles and Kilometres

Notes and Guidance

Children need to know that 5 miles is approximately equal to 8 km. They should use this fact to find approximate conversions from miles to km and from km to miles.

They should be taught the meaning of the symbol '≈' as "is approximately equal to".

Mathematical Talk

Give an example of a length you would measure in miles or km.

If we know 5 miles ≈ 8 km, how can we work out 15 miles converted to km?

Can you think of a situation where you may need to convert between miles and kilometres?

Varied Fluency

5 miles ≈ 8 kilometres

Use this fact to complete:

- 15 miles ≈ _____ km
- 30 miles ≈ _____ km
- _____ miles ≈ 160 km

If 10 miles is approximately 16 km, 1 mile is approximately how many kilometres?

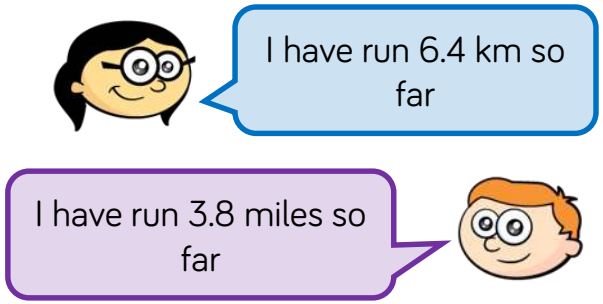
- 2 miles ≈ _____ km
- 4 miles ≈ _____ km
- 0.5 miles ≈ _____ km

In the United Kingdom, the maximum speed on a motorway is 70 miles per hour (mph).
In France, the maximum speed on a motorway is 130 kilometres per hour (km/h).
Which country has the higher speed limit, and by how much? Give your answer in both units.

Miles and Kilometres

Reasoning and Problem Solving

Ron and Annie are running a 5 mile race.



Who has the furthest left to run?

Annie has 1 mile left to run, whereas Ron has 1.2 miles left to run.
Ron has the furthest left to run.

The distance between Cardiff and London is 240 km.

A car is travelling at 60 mph.

How long will it take them to get to London from Cardiff?

240 km ≈ 150 miles
$150 \div 60 = 2\frac{1}{2}$ hours
Or
60 miles ≈ 96 km
$240 \div 96 = 2\frac{1}{2}$ hours

Mo cycles 45 miles over the course of 3 days.

On day 1, he cycles 16 km.

On day 2, he cycles 10 miles further than he did on day 1

How far does he cycle on day 3?

Give your answer in miles and in kilometres.

On day 1 he cycles 16 km / 10 miles.

On day 2 he cycles 32 km / 20 miles.

On day 3 he cycles 24 km / 15 miles.

Imperial Measures

Notes and Guidance

Children need to know and use the following facts:

- 1 foot is equal to 12 inches
- 1 pound is equal to 16 ounces
- 1 stone is equal to 14 pounds
- 1 gallon is equal to 8 pints
- 1 inch is approximately 2.5 cm

They should use these to perform related conversions, both within imperial measures and between imperial and metric.

Mathematical Talk

Put these in order of size: 1 cm, 1 mm, 1 inch, 1 foot, 1 metre. How do you know?

When do we use imperial measures instead of metric measures?

Why are metric measures easier to convert than imperial measures?

Varied Fluency

2.5 cm ≈ 1 inch

1 foot = 12 inches

Use these facts to complete:

2 feet = ____ inches

6 inches ≈ ____ cm

____ feet = 36 inches

4 feet ≈ ____ cm

1 pound (lb) = 16 ounces

1 stone = 14 pounds (lbs)

Use this fact to complete:

2 lbs = ____ ounces

5 stone = ____ lbs

____ lbs = 320 ounces

____ stones = 154 lbs

1 gallon = 8 pints

- How many gallons are equivalent to 64 pints?
- How many pints are equivalent to 15 gallons?
- How many gallons are equivalent to 2 pints?

Imperial Measures

Reasoning and Problem Solving

Jack is 6 foot 2 inches tall. Rosie is 162 cm tall. Who is taller and by how much?	Jack is 185 cm tall, he is 23 cm taller than Rosie.
60 gallons of water are drunk at a sports day. Each child drank 3 pints. How many children were at the sports day?	60 gallons = 480 pints 480 ÷ 3 = 160 children

Eva wants to make a cake. Here are some of the ingredients she needs: • 8 ounces of caster sugar • 6 ounces of self-raising flour • 6 ounces of butter This is what Eva has in her cupboards: • 0.5 lbs of caster sugar • 0.25 lbs of self-raising flour • $\frac{3}{8}$ lbs of butter Does Eva have enough ingredients to bake the cake? If not, how much more does she need to buy? 	Eva has the exact amount of butter and caster sugar, but does not have enough self-raising flour – she needs another 2 ounces.

White Rose Maths

Spring - Block 5

Area, Perimeter & Volume

Overview

Small Steps

- Shapes – same area
- Area and perimeter
- Area of a triangle (1)
- Area of a triangle (2)
- Area of a triangle (3)
- Area of parallelogram
- Volume – counting cubes
- Volume of a cuboid

NC Objectives

Recognise that shapes with the same areas can have different perimeters and vice versa.

Recognise when it is possible to use formulae for area and volume of shapes.

Calculate the area of parallelograms and triangles.

Calculate, estimate and compare volume of cubes and cuboids using standard units, including cm^3, m^3 and extending to other units (mm^3, km^3)

Shapes – Same Area

Notes and Guidance

Children will find and draw rectilinear shapes that have the same area.

Children will use their knowledge of factors to draw rectangles with different areas. They will make connections between side lengths and factors.

Mathematical Talk

What do we need to know in order to work out the area of a shape?

Why is it useful to know your times-tables when calculating area?

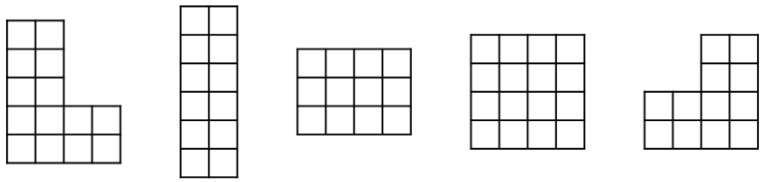
Can you have a square with an area of 48 cm^2? Why?

How can factors help us draw rectangles with a specific area?

Varied Fluency

- Sort the shapes into the Carroll diagram.

	Quadrilateral	Not a quadrilateral
Area of 12 cm^2		
Area of 16 cm^2		



Now draw another shape in each section of the diagram.

- How many rectangles can you draw with an area of 24 cm^2 where the side lengths are integers?

What do you notice about the side lengths?

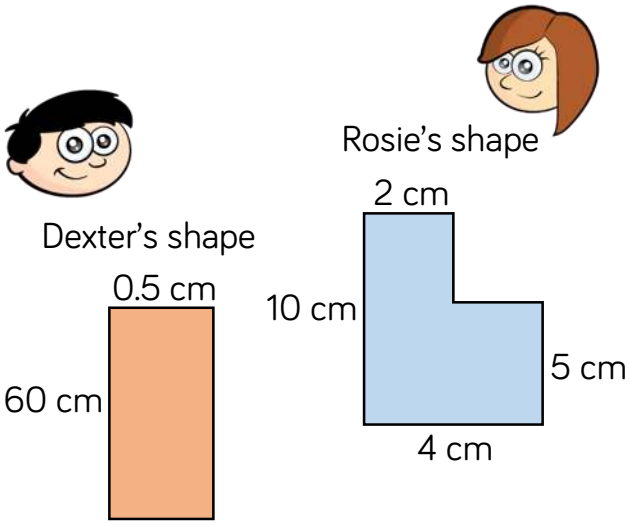
Using integer side lengths, draw as many rectangles as possible that give the following areas:

17 cm^2 25 cm^2 32 cm^2

Shapes – Same Area

Reasoning and Problem Solving

Rosie and Dexter are drawing shapes with an area of 30cm²



Who is correct?

Explain your reasoning.

Both are correct.

Dexter's shape:
60 cm × 0.5 cm
= 30 cm²

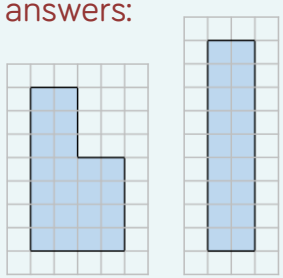
Rosie's shape:
2 cm × 10 cm
= 20 cm²
5 cm × 2 cm
= 10 cm²
20 cm² + 10 cm²
= 30 cm²
Could be split differently.

Three children are given the same rectilinear shape to draw.

Amir says, "The smallest length is 2 cm."
Alex says, "The area is less than 30 cm²."
Annie says, "The perimeter is 22 cm."

What could the shape be?
How many possibilities can you find?

Children can use squared paper to explore. Possible answers:



Always, Sometimes, Never?

If the area of a rectangle is odd then all of the lengths are odd.

Sometimes –
15 cm² could be 5 cm and 3 cm or 60 cm and 0.25 cm

Area and Perimeter

Notes and Guidance

Children should calculate area and perimeter of rectilinear shapes. They must have the conceptual understanding of the formula for area by linking this to counting squares. Writing and using the formulae for area and perimeter is a good opportunity to link back to the algebra block.
Children explore that shapes with the same area can have the same or different perimeters.

Mathematical Talk

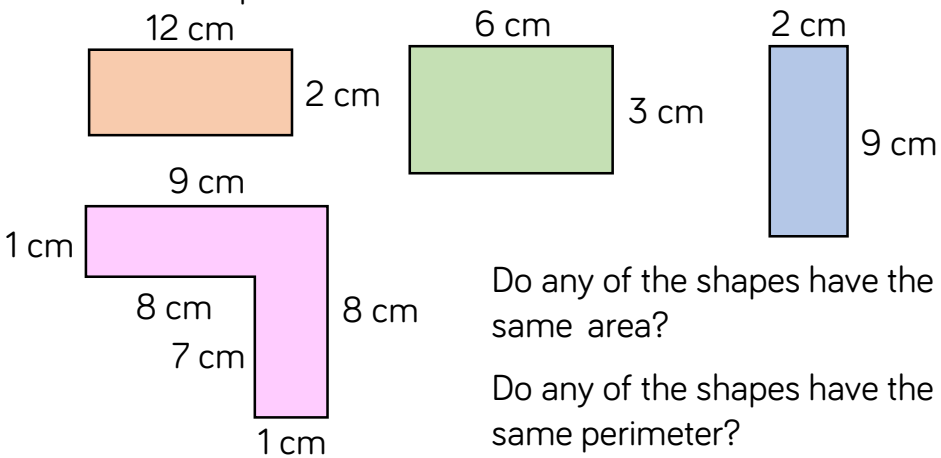
What is the difference between the area and perimeter of a shape?

How do we work out the area and perimeter of shapes? Can you show this as a formula?

Can you have 2 rectangles with an area of 24 cm² but different perimeters?

Varied Fluency

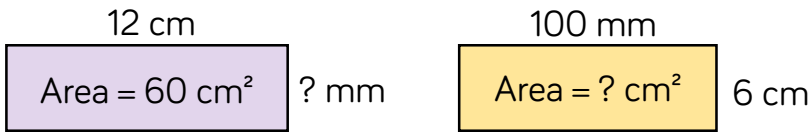
- Look at the shapes below.



Do any of the shapes have the same area?

Do any of the shapes have the same perimeter?

- Work out the missing values.



- Draw two rectilinear shapes that have an area of 36 cm² but have different perimeters.

What is the perimeter of each shape?

Area and Perimeter

Reasoning and Problem Solving

True or false?

Two rectangles with the same perimeter can have different areas.

Explain your answer.

True. Children explore this by drawing rectangles and comparing both area and perimeter.

A farmer has 60 metres of perimeter fencing.

For every 1 m² he can keep 1 chicken.

How can he arrange his fence so that the enclosed area gives him the greatest area?

The greatest area is a 15 m × 15 m square, giving 225 m²

Children may create rectangles by increasing one side by 1 unit and decreasing one side by 1 unit e.g.

16 × 14 = 224 m²

17 × 13 = 221 m²

Tommy has a 8 cm × 2 cm rectangle. He increases the length and width by 1 cm.

Length	Width	Area
8	2	
9	3	

He repeats with a 4 cm × 6 cm rectangle.

Length	Width	Area
4	6	

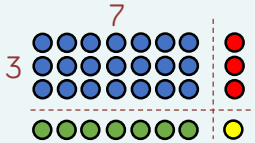
What do you notice happens to the areas?

Can you find any other examples that follow this pattern?

Are there any examples that do not follow the pattern?

If the sum of the length and width is 10, then the area will always increase by 11

Children may use arrays to explore this:



The red and green will always total 10 and the yellow will increase that by 1 to 11

Area of a Triangle (1)

Notes and Guidance

Children will use their previous knowledge of approximating and estimating to work out the area of different triangles by counting.
Children will need to physically annotate to avoid repetition when counting the squares.
Children will begin to see the link between the area of a triangle and the area of a rectangle or square.

Mathematical Talk

How many whole squares can you see?

How many part squares can you see?

What could we do with the parts?

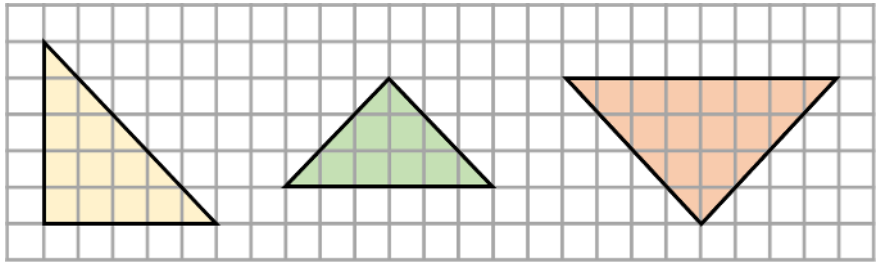
What does estimate mean?

Why is your answer to this question an **estimate** of the area?

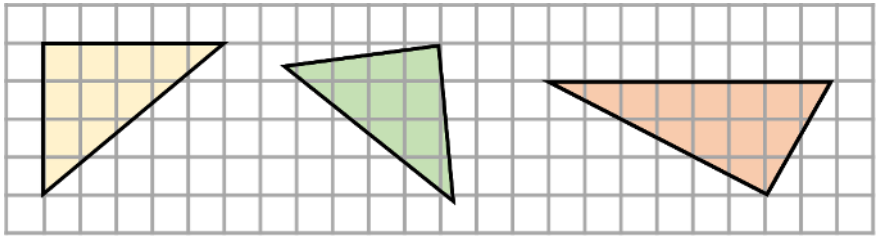
Revisit the idea that a square is a rectangle when generalising how to calculate the area of a triangle.

Varied Fluency

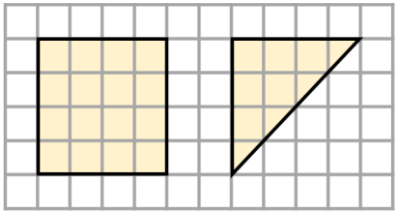
- Count squares to calculate the area of each triangle.



- Estimate the area of each triangle by counting squares.



- Calculate the area of each shape by counting squares.

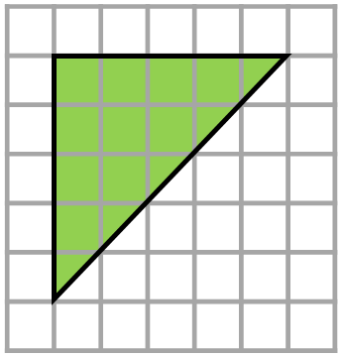


What do you notice about the area of the triangle compared to the area of the square?
Does this always happen?

Explore this using different rectangles.

Area of a Triangle (1)

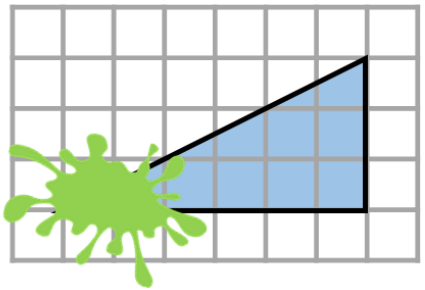
Reasoning and Problem Solving



Mo says the area of this triangle is 15cm²
Is Mo correct? If not, explain his mistake.

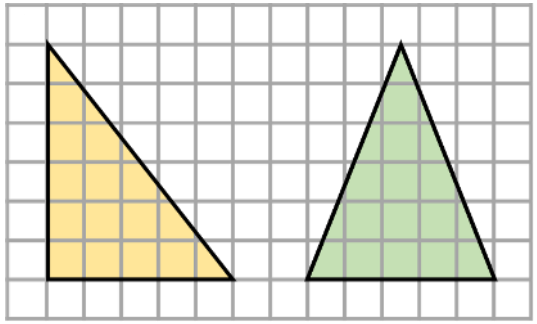
Mo is incorrect because he has counted the half squares as whole squares.

Part of a triangle has been covered.
Estimate the area of the whole triangle.



9 cm²

What is the same about these two triangles?
What is different?



Can you create a different right angled triangle with the same area?

Both triangles have an area of 15 cm²
The triangle on the left is a right angled triangle and the triangle on the right is an isosceles triangle.

Children could draw a triangle with a height of 10 cm and a base of 3 cm, or a height of 15 cm and a base of 2 cm.

Area of a Triangle (2)

Notes and Guidance

Children use their knowledge of finding the area of a rectangle to find the area of a right-angled triangle. They see that a right-angled triangle with the same length and perpendicular height as a rectangle will have an area half the size.

Using the link between the area of a rectangle and a triangle, children will learn and use the formula to calculate the area of a triangle.

Mathematical Talk

What is the same/different about the rectangle and triangle?

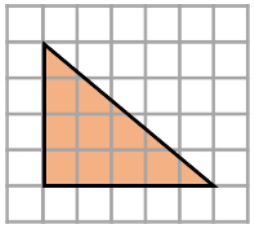
What is the relationship between the area of a rectangle and the area of a right-angled triangle?

What is the formula for working out the area of a rectangle or square?

How can you use this formula to work out the area of a right-angled triangle?

Varied Fluency

- Estimate the area of the triangle by counting the squares.
  Make the triangle into a rectangle with the same height and width. Calculate the area.

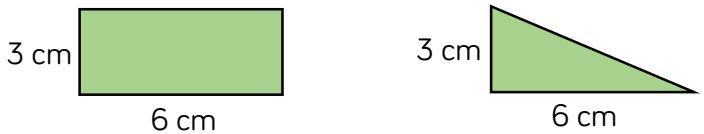


  The area of the triangle is _____ the area of the rectangle.

- If l represents length and h represents height:

  Area of a rectangle $= l \times h$

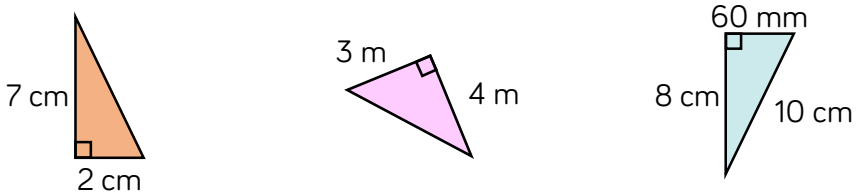
  Use this to calculate the area of the rectangle.



  What do you need to do to your answer to work out the area of the triangle?

  Therefore, what is the formula for the area of a triangle?

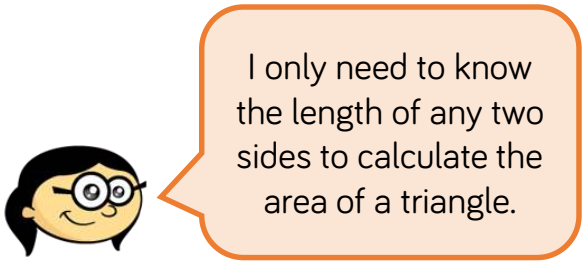
- Calculate the area of these triangles.



Area of a Triangle (2)

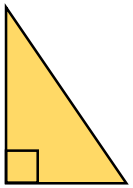
Reasoning and Problem Solving

Annie is calculating the area of a right-angled triangle.



Do you agree with Annie? Explain your answer.

Annie is incorrect as it is not sufficient to know **any** two sides, she needs the base and perpendicular height. Children could draw examples and non-examples.



Area = 54 cm²

What could the length and the height of the triangle be?

How many different integer possibilities can you find?

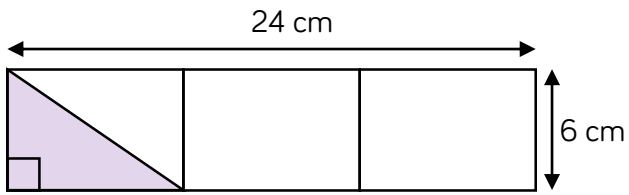
Possible answers:

Height: 18 cm
Base: 6 cm

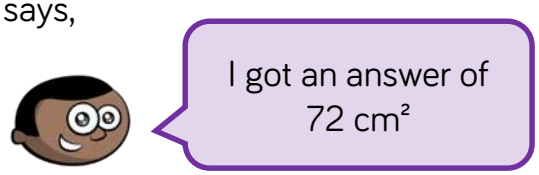
Height: 27 cm
Base: 4 cm

Height: 12 cm
Base: 9 cm

Calculate the area of the shaded triangle.



Mo says,



Do you agree with Mo?
If not, can you spot his mistake?

The area of the shaded triangle is 24 cm²

Mo is incorrect as he has just multiplied the two numbers given and divided by 2, he hasn't identified the correct base of the triangle.

Area of a Triangle (3)

Notes and Guidance

Children will extend their knowledge of working out the area of a right-angled triangle to work out the area of any triangle.

They use the formula, base × perpendicular height ÷ 2 to calculate the area of a variety of triangles where different side lengths are given and where more than one triangle make up a shape.

Mathematical Talk

What does the word perpendicular mean?

What do we mean by perpendicular height?

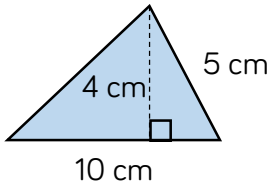
What formula can you use to calculate the area of a triangle?

If there is more than one triangle making up a shape, how can we use the formula to find the area of the whole shape?

How do we know which length tells us the perpendicular height of the triangle?

Varied Fluency

- To calculate the height of a triangle, you can use the formula:
  base × height ÷ 2
  Choose the correct calculation to find the area of the triangle.

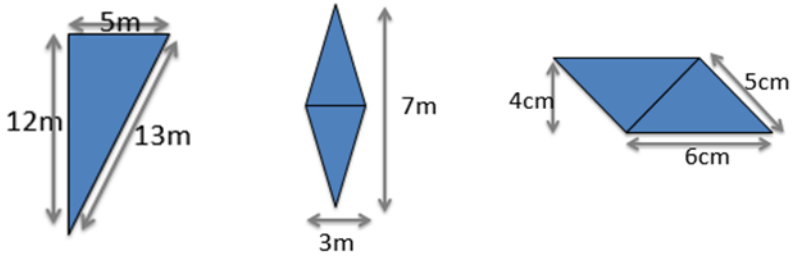


- 10 × 5 ÷ 2
- 10 × 4 ÷ 2
- 5 × 4 ÷ 2

- Estimate the area of the triangle by counting squares.

Now calculate the area of the triangle.
Compare your answers.

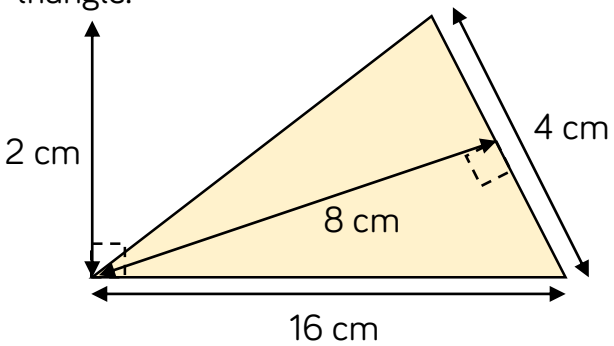
- Calculate the area of each shape.



Area of a Triangle (3)

Reasoning and Problem Solving

Class 6 are calculating the area of this triangle.



Here are some of their methods.

4 × 8 × 16 × 2 ÷ 2	4 × 8 ÷ 2
16 × 2 ÷ 2	16 × 4 ÷ 2
16 × 8 ÷ 2	8 × 1

Tick the correct methods.

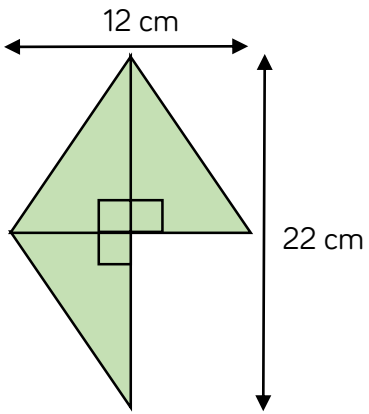
Explain any mistakes.

The correct methods are:
16 × 2 ÷ 2
4 × 8 ÷ 2

All mistakes are due to not choosing a pair of lengths that are perpendicular.

Children could explore other methods to get to the correct answer e.g. halving the base first and calculating 8 × 2 etc.

The shape is made of three identical triangles.



What is the area of the shape?

Each triangle is 6 cm by 11 cm so area of one triangle is 33 cm²

Total area = 99 cm²

Area of a Parallelogram

Notes and Guidance

Children use their knowledge of finding the area of a rectangle to find the area of a parallelogram.

Children investigate the link between the area of a rectangle and parallelogram by cutting a parallelogram so that it can be rearranged into a rectangle. This will help them understand why the formula to find the area of parallelograms works.

Mathematical Talk

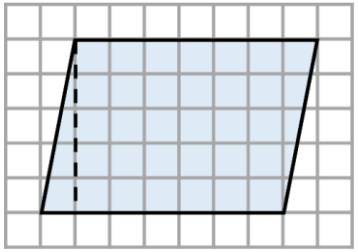
Describe a parallelogram.

What do you notice about the area of a rectangle and a parallelogram?

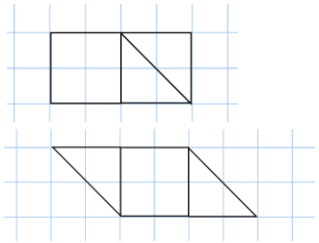
What formula can you use to work out the area of a parallelogram?

Varied Fluency

- Approximate the area of the parallelogram by counting squares.
  Now cut along the dotted line.
  Can you move the triangle to make a rectangle?
  Calculate the area of the rectangle.

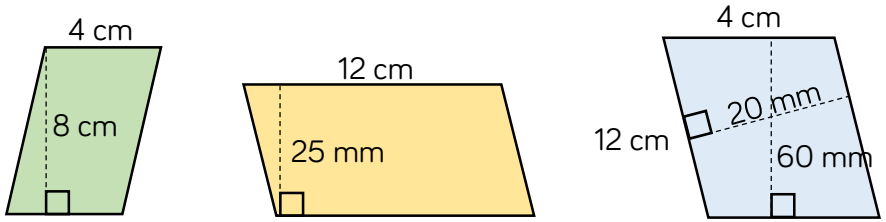


- Here are two quadrilaterals.



  - What is the same about the quadrilaterals?
  - What's different?
  - What is the area of each quadrilateral?

- Use the formula base × perpendicular height to calculate the area of the parallelograms.



Area of a Parallelogram

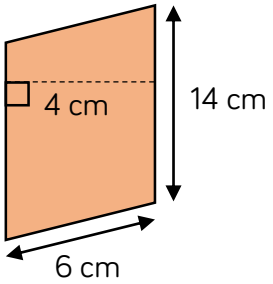
Reasoning and Problem Solving

Teddy has drawn a parallelogram.

The area is greater than 44 m² but less than 48 m².

What could the base length and the perpendicular height of Teddy's parallelogram be?

Possible answers:
9 m by 5 m
= 45 m²
6.5 m by 7 m
= 45.5 m²
11 m by 4.2 m
= 46.2 m²



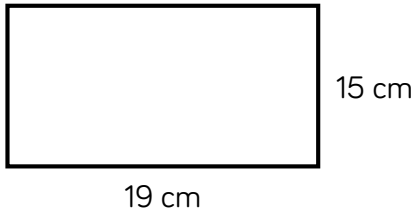
Dexter thinks the area of the parallelogram is 84 cm².

What mistake has Dexter made?

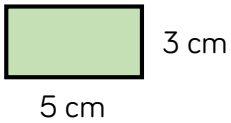
What is the correct area?

Dexter has multiplied 14 by 6 when he should have multiplied by 4 because 4 is the perpendicular height of the parallelogram.
The correct area is 56 cm².

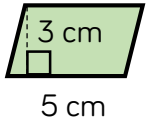
Dora and Eva are creating a mosaic.
They are filling a sheet of paper this size.



Dora is using tiles that are rectangular.



Eva's tiles are parallelograms.



Dora thinks that she will use fewer tiles than Eva to fill the page because her tiles are bigger.
Do you agree? Explain your answer.

Dora is wrong because both hers and Eva' s tiles have the same area and so the same number of tiles will be needed to complete the mosaic.
The area of the paper is 285 cm² and the area of each tile is 15 cm² so 19 tiles are needed to complete the pattern.

Volume – Counting Cubes

Notes and Guidance

Children should understand that volume is the space occupied by a 3-D object.

Children will start by counting cubic units (1 cm^3) to find the volume of 3D shapes. They will then use cubes to build their own models and describe the volume of the models they make.

Mathematical Talk

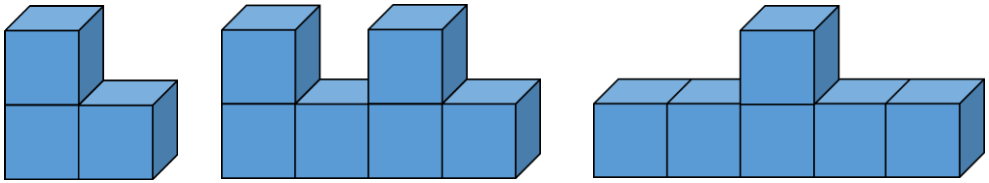
What's the same and what's different between area and volume?

Can you explain how you worked out the volume?
What did you visualise?

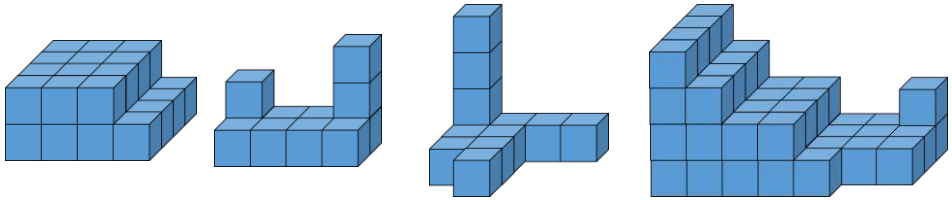
What units of measure could we use for volume? (Explore cm^3, m^3, mm^3 etc.)

Varied Fluency

- If each cube has a volume of 1 cm^3, find the volume of each solid.



- Make each shape using multilink cubes.



If each cube has a volume of 1 cm^3, what is the volume of each shape?
Place the shapes in ascending order based on their volume.
What about if each cube represented 1 mm^3, how would this affect your answer? What about if they were 1 m^3?

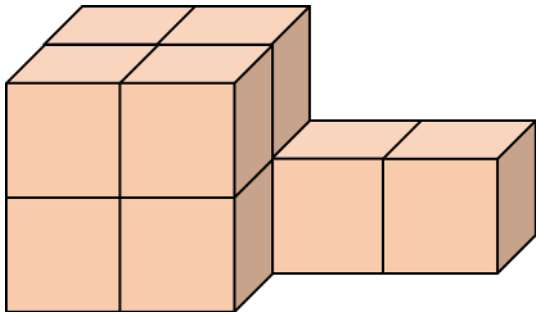
- If one multilink cube represents 1 cubic unit, how many different models can you make with a volume of 12 cubic units?

Volume – Counting Cubes

Reasoning and Problem Solving

Amir says he will need 8 cm^3 to build this shape.

Dora says she will need 10 cm^3.



Who do you agree with?

Explain why.

Amir is incorrect because he has missed the 2 cubes that cannot be seen.
Dora is correct because there are 8 cm^3 making the visible shape, then there are an additional 2 cm^3 behind.

Tommy is making cubes using multilink. He has 64 multilink cubes altogether.

How many different sized cubes could he make?

He says,

If I use all of my multilink to make 8 larger cubes, then each of these will be 2 by 2 by 2.

How many other combinations can Tommy make where he uses all the cubes?

Tommy could make:

- $1 \times 1 \times 1$
- $2 \times 2 \times 2$
- $3 \times 3 \times 3$
- $4 \times 4 \times 4$

Possible answers:

64 cubes that are $1 \times 1 \times 1$

2 cubes that are $3 \times 3 \times 3$; 1 cube that is $2 \times 2 \times 2$; 2 cubes that are $1 \times 1 \times 1$

Volume of a Cuboid

Notes and Guidance

Children make the link between counting cubes and the formula ($l \times w \times h$) for calculating the volume of cuboids.

They realise that the formula is the same as calculating the area of the base and multiplying this by the height.

Mathematical Talk

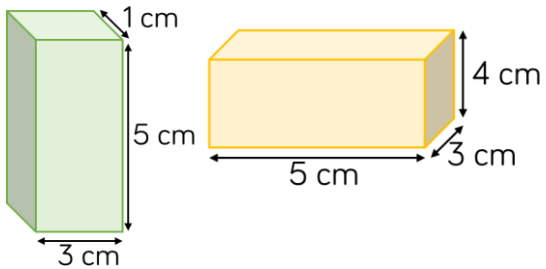
Can you identify the length, width and height of the cuboid?

If the length of a cuboid is 5 cm and the volume is 100 cm³, what could the width and height of the cuboid be?

What knowledge can I use to help me calculate the missing lengths?

Varied Fluency

- Complete the sentences for each cuboid.



The length is: _____
The width is: _____
The height is: _____

The area of the base is: ____ × ____ = ____
Volume = The area of the base × ____ = ____

- Calculate the volume of a cube with side length:

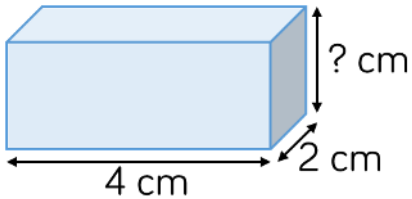
4 cm 2 m 160 mm

Use appropriate units for your answers.

- The volume of the cuboid is 32 cm³.

Calculate the height.

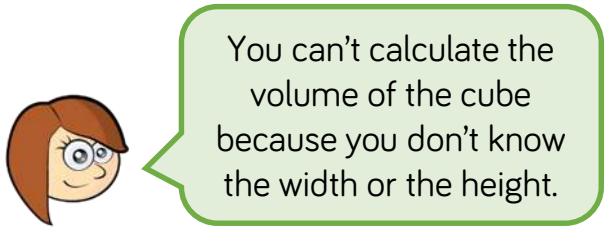
You might want to use multilink cubes to help you.

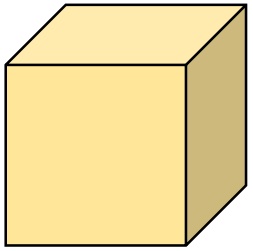


Volume of a Cuboid

Reasoning and Problem Solving

Rosie says,





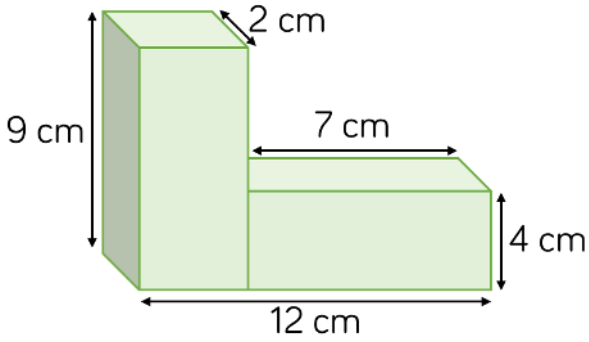
Do you agree?

Explain why.

You don't need the rest of the measurements because it's a cube and all the edges of a cube are equal.
Therefore, the width would be 2 cm and the height would be 2 cm.

The volume of the cube is 8 cm³

Calculate the volume of the shape.



146 cm³

How many different ways can you make a cuboid with a volume of 48 cm³?

Possible answers:

24 × 2 × 1
2 × 6 × 4
6 × 8 × 1

White Rose Maths

Spring - Block 6

Ratio

Overview

Small Steps

- Using ratio language
- Ratio and fractions
- Introducing the ratio symbol
- Calculating ratio
- Using scale factors
- Calculating scale factors
- Ratio and proportion problems

NC Objectives

Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.

Solve problems involving similar shapes where the scale factor is known or can be found.

Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

Using Ratio Language

Notes and Guidance

Children will understand that a ratio shows the relationship between two values and can describe how one is related to another.

They will start by making simple comparisons between two different quantities. For example, they may compare the number of boys to girls in the class and write statements such as, “For every one girl, there are two boys”.

Mathematical Talk

How would your sentences change if there were 2 more blue flowers?

How would your sentences change if there were 10 more pink flowers?

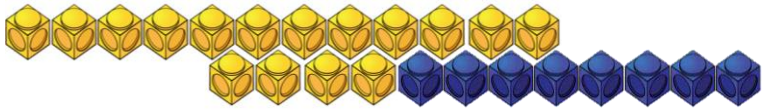
Can you write a “For every…” sentence for the number of boys and girls in your class?

Varied Fluency

- Complete the sentences.

For every two blue flowers there are ____ pink flowers.
For every blue flower there are _____ pink flowers.

- Use cubes to help you complete the sentences.



For every ____ , there are ____

For every 8 , there are ____

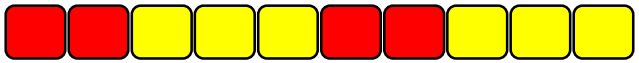
For every 1 , there are ____

- How many “For every…” sentences can you write to describe these counters?

Using Ratio Language

Reasoning and Problem Solving

Whitney lays tiles in the following pattern



If she has 16 red tiles and 20 yellow tiles remaining, can she continue her pattern without there being any tiles left over?

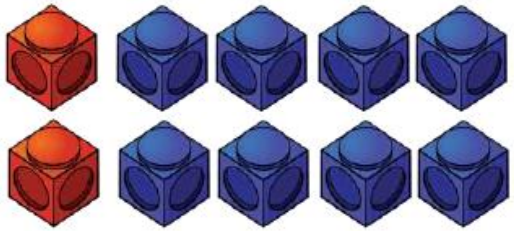
Explain why.

Possible responses:

For every two red tiles there are three yellow tiles. If Whitney continues the pattern she will need 16 red tiles and 24 yellow tiles. She cannot continue the pattern without there being tiles left over.

20 is not a multiple of 3

True or False?



- For every red cube there are 8 blue cubes. — False
- For every 4 blue cubes there is 1 red cube. — True
- For every 3 red cubes there would be 12 blue cubes. — True
- For every 16 cubes, 4 would be red and 12 would be blue. — False
- For every 20 cubes, 4 would be red and 16 would be blue. — True

Ratio and Fractions

Notes and Guidance

Children often think a ratio 1 : 2 is the same as a fraction of $\frac{1}{2}$

In this step, they use objects and diagrams to compare ratios and fractions.

Mathematical Talk

How many counters are there altogether?

How does this help you work out the fraction?

What does the denominator of the fraction tell you?

How can a bar model help you to show the mints and chocolates?

Varied Fluency

- The ratio of red counters to blue counters is 1 : 2

What fraction of the counters is blue? $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$

What fraction of the counters is red? $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$

- This bar model shows the ratio 2 : 3 : 4

What fraction of the bar is pink?
What fraction of the bar is yellow?
What fraction of the bar is blue?

- One third of the sweets in a box are mints.
The rest are chocolates.
What is the ratio of mints to chocolates in the box?

Ratio and Fractions

Reasoning and Problem Solving

Ron plants flowers in a flower bed.
For every 2 red roses he plants 5 white roses.

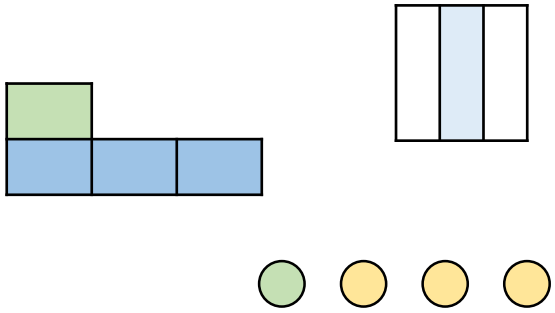
He says,

$\frac{2}{5}$ of the roses are red.

Is Ron correct?

Ron is incorrect because $\frac{2}{7}$ of the roses are red. He has mistaken a part with the whole.

Which is the odd one out?
Explain your answer.



is the odd one out because one part out of three is a different colour. The others are one part out of four.

There are some red and green cubes in a bag. $\frac{2}{5}$ of the cubes are red.

True or False?

- For every 2 red cubes there are 5 green cubes. — False
- For every 2 red cubes there are 3 green cubes. — True
- For every 3 green cubes there are 2 red cubes. — True
- For every 3 green cubes there are 5 red cubes. — False

Explain your answers.

Introducing the Ratio Symbol

Notes and Guidance

Children are introduced to the colon notation as the ratio symbol, and continue to link this with the language 'for every…, there are…'
They need to read ratios e.g. 3 : 5 as "three to five".
Children understand that the notation relates to the order of parts. For example, 'For every 3 bananas there are 2 apples would be the same as 3 : 2 and for every 2 apples there are 3 bananas would be the same as 2 : 3

Mathematical Talk

What does the : symbol mean in the context of ratio?

Why is the order of the numbers important when we write ratios?

How do we write a ratio that compares three quantities?

How do we say the ratio "3 : 7"?

Varied Fluency

- Complete:

The ratio of red counters to blue counters is ☐ : ☐

The ratio of blue counters to red counters is ☐ : ☐

- Write down the ratio of:

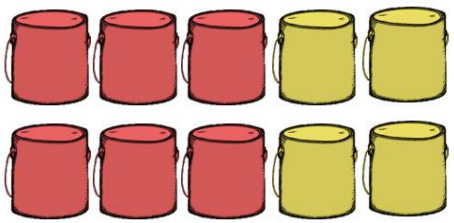
  - Bananas to strawberries
  - Blackberries to strawberries
  - Strawberries to bananas to blackberries
  - Blackberries to strawberries to bananas

- The ratio of red to green marbles is 3 : 7
  Draw an image to represent the marbles.
  What fraction of the marbles are red?
  What fraction of the marbles are green?

Introducing the Ratio Symbol

Reasoning and Problem Solving

Tick the correct statements.



- There are two yellow tins for every three red tins.
- There are two red tins for every three yellow tins.
- The ratio of red tins to yellow tins is 2 : 3
- The ratio of yellow tins to red tins is 2 : 3

Explain which statements are incorrect and why.

The first and last statement are correct. The other statements have the ratios the wrong way round.

In a box there are some red, blue and green pens.

The ratio of red pens to green pens is 3 : 5

For every 1 red pen there are two blue pens.

Write down the ratio of red pens to blue pens to green pens.

R : G

3 : 5

R : B

1 : 2 or

3 : 6

R : B : G

3 : 6 : 5

Calculating Ratio

Notes and Guidance

Children build on their knowledge of ratios and begin to calculate ratios. They answer worded questions in the form of 'for every… there are …' and need to be able to find both a part and a whole.
They should be encouraged to draw bar models to represent their problems, and clearly label the information they have been given and what they want to calculate.

Mathematical Talk

How can we represent this ratio using a bar model?

What does each part represent? What will each part be worth?

How many parts are there altogether? What is each part worth?

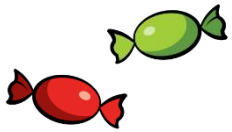
If we know what one part is worth, can we calculate the other parts?

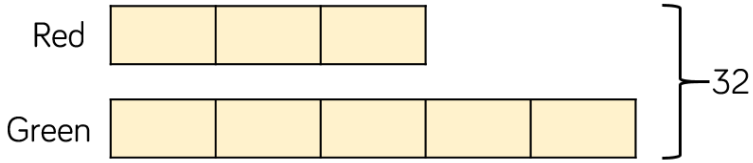
Varied Fluency

- A farmer plants some crops in a field.
  For every 4 carrots he plants 2 leeks.
  He plants 48 carrots in total.
  How many leeks did he plant?
  How many vegetables did he plant in total?

- Jack mixes 2 parts of red paint with 3 parts blue paint to make purple paint.
  If he uses 12 parts blue paint, how many parts red paint does he use?

- Eva has a packet of sweets.
  For every 3 red sweets there are 5 green sweets.
  If there are 32 sweets in the packet in total, how many of each colour are there?
  You can use a bar model to help you.





Calculating Ratio

Reasoning and Problem Solving

Teddy has two packets of sweets.

In the first packet, for every one strawberry sweet there are two orange sweets.

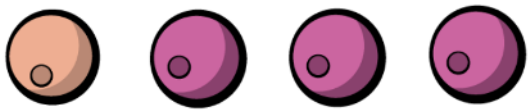
In the second packet, for every three orange sweets there are two strawberry sweets.

Each packet contains 15 sweets in total.

Which packet has more strawberry sweets and by how many?

The first packet has 5 strawberry sweets and 10 orange sweets.
The second packet has 6 strawberry sweets and 9 orange sweets.
The second packet has 1 more strawberry sweet than the first packet.

Annie is making some necklaces to sell.
For every one pink bead, she uses three purple beads.



Each necklace has 32 beads in total.

The cost of the string is £2.80
The cost of a pink bead is 72p.
The cost of a purple bead is 65p.

How much does it cost to make one necklace?

Each necklace has 8 pink beads and 24 purple beads.

The cost of the pink beads is £5.76

The cost of the purple beads is £15.60

The cost of a necklace is £24.16

Using Scale Factors

Notes and Guidance

In this step, children enlarge shapes to make them 2 or 3 times as big etc. They need to be introduced to the term "scale factor" as the name for this process.

Children should be able to draw 2-D shapes on a grid to a given scale factor and be able to use vocabulary, such as, "Shape A is three times as big as shape B".

Mathematical Talk

What does enlargement mean?

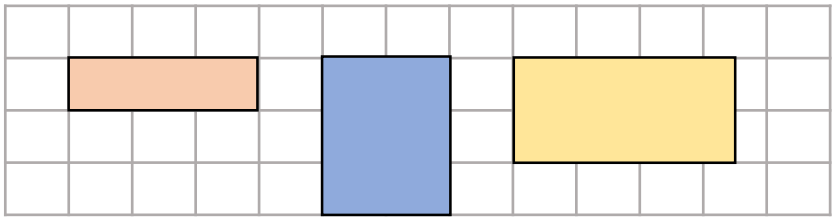
What does scale factor mean?

Why do we have to double/triple all the sides of each shape?

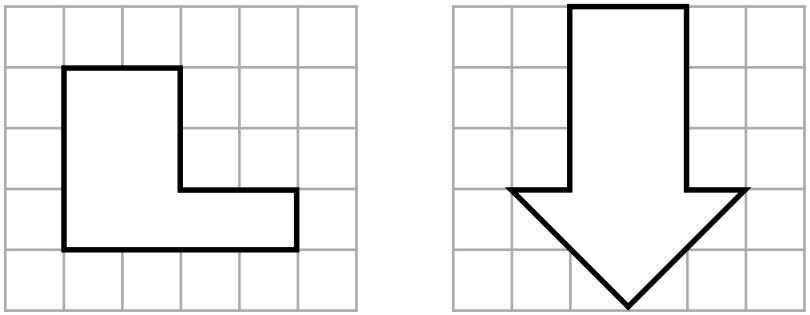
Have the angles changed size?

Varied Fluency

- Copy these rectangles onto squared paper then draw them double the size, triple the size and 5 times as big.



- Copy these shapes onto squared paper then draw them twice as big and three times as big.



- Enlarge these shapes by:
  - Scale factor 2
  - Scale factor 3
  - Scale factor 4

Using Scale Factors

Reasoning and Problem Solving

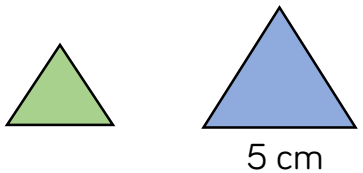
Draw a rectangle 3 cm by 4 cm.

Enlarge your rectangle by scale factor 2.

Compare the perimeter, area and angles of your two rectangles.

The perimeter has doubled, the area is four times as large, the angles have stayed the same.

Here are two equilateral triangles.
The blue triangle is three times larger than the green triangle.



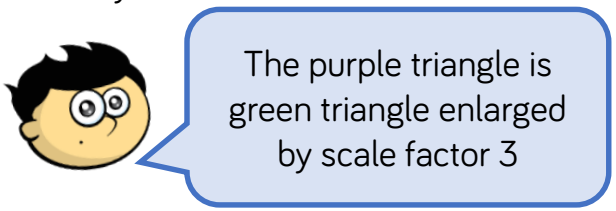
(Not drawn to scale)

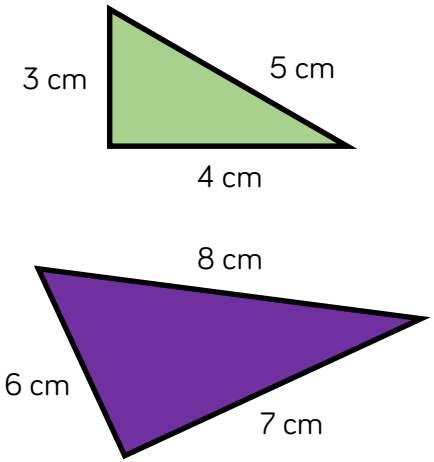
Find the perimeter of both triangles.

The blue triangle has a perimeter of 15 cm.

The green triangle has a perimeter of 5 cm.

Jack says:





Do you agree?
Explain why.

Possible answer
I do not agree because Jack has increased the green shape by adding 3 cm to each side, not increasing it by a scale factor of 3

Calculating Scale Factors

Notes and Guidance

Children find scale factors when given similar shapes. They need to be taught that 'similar' in mathematics means that one shape is an exact enlargement of the other, not just they have some common properties.

Children use multiplication and division facts to calculate missing information and scale factors.

Mathematical Talk

What does similar mean?

What do you notice about the length/width of each shape?

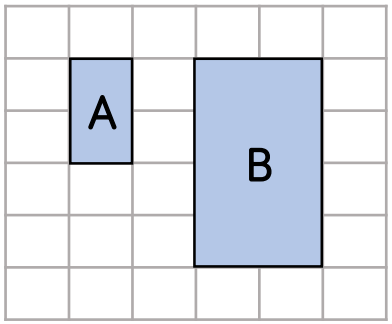
How would drawing the rectangles help you?

How much larger/smaller is shape A compared to shape B?

What does a scale factor of 2 mean? Can you have a scale factor of 2.5?

Varied Fluency

- Complete the sentences.



Shape B is _____ as big as shape A.

Shape A has been enlarged by scale factor _____ to make shape B.

- The rectangles described in the table are all similar to each other. Fill in the missing lengths and widths and complete the sentences.

Rectangle	Length	Width
A	5 cm	2 cm
B		4 cm
C	25 cm	
D		18 cm

From A to B, the scale factor of enlargement is ____
From A to C, the scale factor of enlargement is ____
From A to D the scale factor of enlargement is ____
From B to D, the scale factor of enlargement is ____

Calculating Scale Factors

Reasoning and Problem Solving

A rectangle has a perimeter of 16 cm. An enlargement of this rectangle has a perimeter of 24 cm.

The length of the smaller rectangle is 6 cm.

Draw both rectangles.

Smaller rectangle:
length – 6 cm
width – 2 cm

Larger rectangle:
length – 9 cm
width – 3 cm

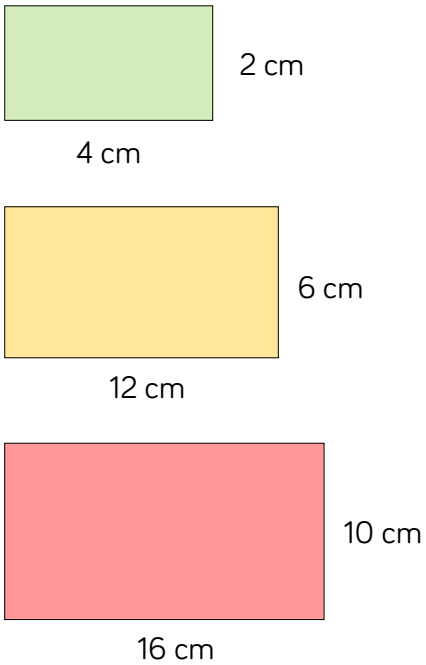
Scale factor: 1.5

Always, sometimes, or never true?

To enlarge a shape you just need to do the same thing to each of the sides.

Sometimes.
This only works when we are multiplying or dividing the lengths of the sides. It does not work when adding or subtracting.

Ron says that these three rectangles are similar.



Do you agree?
Explain your answer.

Ron is incorrect.
The orange rectangle is an enlargement of the green rectangle with scale factor 3.
The red rectangle, however, is not similar to the other two as the side lengths are not in the same ratio.

Ratio and Proportion Problems

Notes and Guidance

Children will apply the skills they have learnt in the previous steps to a wide range of problems in different contexts.

They may need support to see that different situations are in fact alternative uses of ratio.

Bar models will again provide valuable pictorial support.

Mathematical Talk

How does this problem relate to ratio?

Can we represent this ratio using a bar model?

What does each part represent? What is the whole?

What is the same about the ratios?

What is different about them?

Varied Fluency

- How much of each ingredient is needed to make soup for:
  - 3 people
  - 9 people
  - 1 person

  What else could you work out?

Recipe for 6 people

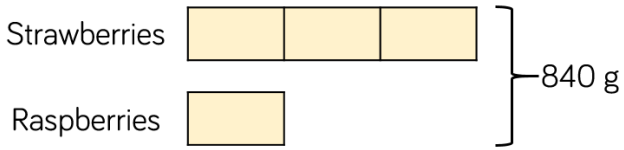
- 1 onion
- 60 g butter
- 180 g lentils
- 1.2 litres stock
- 480 ml tomato juice

- Two shops sell the same pens for these prices.

Safeway	K-mart
4 pens £2.88	7 pens £4.83

Which shop is better value for money?

- The mass of strawberries in a smoothie is three times the mass of raspberries in the smoothie. The total mass of the fruit is 840 g. How much of each fruit is needed.



Ratio and Proportion Problems

Reasoning and Problem Solving

This recipe makes 10 flapjacks.

Flapjacks

120 g butter
100 g brown sugar
4 tablespoons golden syrup
250 g oats
40 g sultanas

Amir has 180 g butter.

What is the largest number of flapjacks he can make?

How much of the other ingredients will he need?

He has enough butter to make 15 flapjacks.
He will need 150 g brown soft sugar, 6 tablespoons golden syrup, 375 g oats and 60 g sultanas.

Alex has two packets of sweets.

In the first packet, for every 2 strawberry sweets there are 3 orange.

In the second packet, for one strawberry sweet, there are three orange.

Each packet has the same number of sweets.

The second packet contains 15 orange sweets.

How many strawberry sweets are in the first packet?

Second packet:
15 orange
5 strawberry.

So there are 20 sweets in each packet.

First packet:
8 strawberry
12 orange

The first packet contains 8 strawberry sweets.